ANALYSIS OF ONE-WAY CONTINUOUS SKEW PLATE-BEAM SYSTEMS BY FLEXIBILITY METHODS

By

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1960

Submitted to the Faculty of the Graduate School of
the Oklahoma State University
in partial fulfillment of the requirements
for the degree of
MASTER OF SCIENCE
August, 1962

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PREFACE

I wish to express my sincere gratitude to the following individuals:

To Dr. Kerry S. Havner, my major advisor, for his help in the selection of the topic of this thesis, and for his valuable guidance and criticism throughout the preparation of the manuscript;

To Prof. R. L. Flanders for acting as my second advisor;

To Prof. Jan J. Tuma for providing an opportunity to study at Oklahoma State University;

To the Faculty of the School of Civil Engineering for their valuable instructions throughout my study;

To Messrs. M. N. Reddy, R. K. Munshi and J. A Patel for their interest in my study;

To my parents for their complete understanding, patience and sacrifices to provide during my study;

Finally, to Mrs. Virginia Schenandoah for her effort in typing the final manuscript very carefully.

J. M. Patel

July, 1962
Stillwater, Oklahoma

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NOMENCLATURE

a, b, c	Carry-Over Factors
d	Dimensionless Quantity
h	Thickness of Plate
i, j, k	Network Points
p	Intensity of Load
t	Dimensionless Quantity
u, v	Skew Co-ordinates, Co-ordinate Axis
x, y	Rectangular Co-ordinates, Co-ordinate Axis
B	Flexural Rigidity of Beam
D	Flexural Rigidity of Plate
D_{ii}	Displacement Flexibility
E	Young's Modulus of Elasticity
F_{ii}	Angular Flexibility
G_{ij}	Angular Carry-Over
H_{ij}	Displacement Carry-Over
M	Sum of Bending Moments $\times \frac{1}{(1+\mu)}$
O	Origin of Co-ordinate System
P	Concentrated Load
Q_{ij}	Angular-Displacement Carry-Over
R	Vertical Edge Reaction Per Unit Length

W	Vertical Deflection
α	Angle of Skew
δ	Displacement Load Function
η	Deflection Influence Coefficient
θ	Angle of Rotation of Plate
λ	Dimensionless Quantity
μ	Poisson's Ratio
τ	Angular Load Function
ΔA	Area of the Domain of Point ij
Δu, Δv	Dimensions of Plate Element
∇^2	Laplace Partial Differential Operator

CHAPTER I

INTRODUCTION

1.1. Historical Review. The problem of continuous skew plates has not received extensive treatment mathematically because of the difficulties in handling the boundary conditions with skew coordinates. A number of papers utilizing numerical methods have been written, however, based on a finite-difference approximation of the governing differential equation. H. Marcus (1)* first applied difference equations to slabs and presented the fundamental equations for the skew network. Cecilia Vittorio Brigatti (2) applied Marcus' difference equations to a skew slab uniformly loaded with all four edges either simply supported or fixed. In 1941, V. P. Jensen (3) developed a procedure for analysis of a single span skew slab-bridge with curbsusing a triangular network. T. Y. Chen, C. P. Siess, and N M. Newmark (4) published a paper based on the theory of continuous isotropic parallelogram plates supported by flexible girders in 1957. Asim Yeginobali (5), in 1958, analyzed a three span continuous skewed slab using V. P. Jensen's difference equations with modification made for the continuity of adajacent spans. He also included tables for the deflections and moments for different types of continuous slabs under various uniform loading conditions. Masao Naruoka and Hiroshi Ohmura (6) derived skew network finite difference equations for the

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*Numbers in parentheses, after names, refer to numbered references in Selected Bibliography.

orthotropic parallelogram plate, and published the influence coefficients for deflection and bending moment for the plate simply supported at two opposite edges, the other two being supported by flexible girders.

1.2. Scope of Study. A method for analyzing a thin continuous skewed slab of constant flexural rigidity is presented. Each span is considered as a basic unit with two parallel edges simply supported and the other two edges free. For each basic span, the middle surface of the slab is covered by a skew finite-difference network. A set of simultaneous equations, each expressing the deflection of a certain point of the network in terms of the surrounding points, is formed. The numerical solution of these equations gives the deflections of all points on the network, which in turn can be used to express the bending moments, twisting moments and shears at any point on the slab.

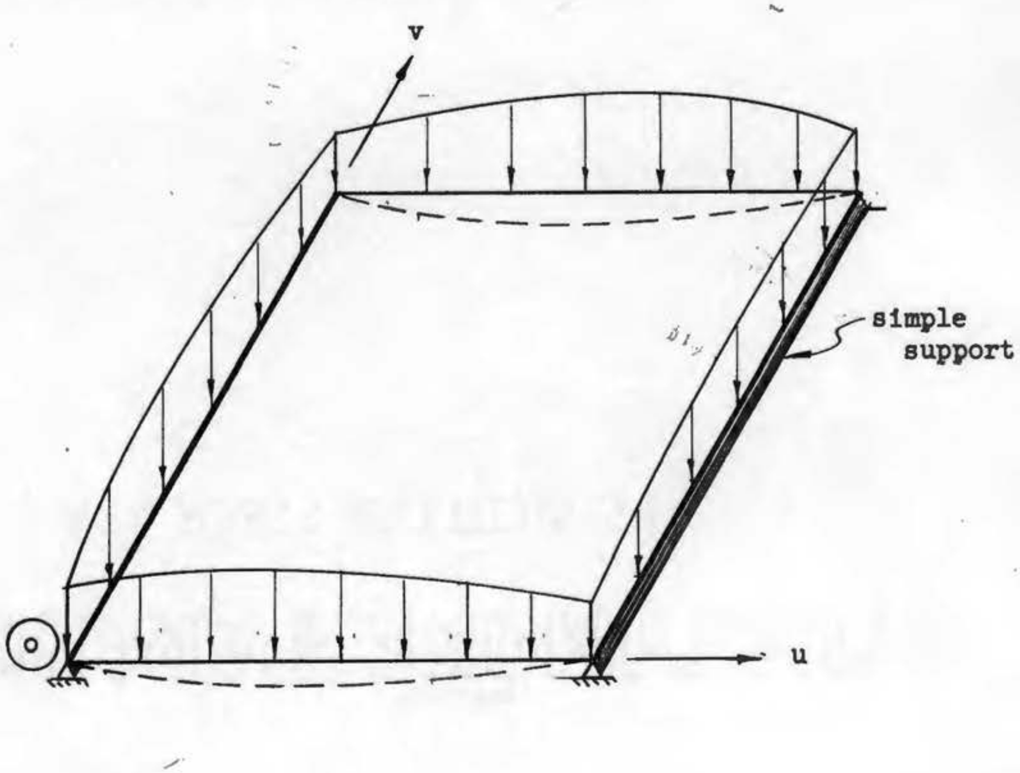

Figure 1.1. Basic Structure

Using the flexibility approach discussed by Tuma (7) in a graduate course in plate structures, the angular functions and the load functions are defined in terms of the influence coefficients for deflection of the simple basic slab. A table for these influence coefficients for one particular problem is presented in this paper.

A general moment-reaction equation in matrix form is derived using the compatibility between adjoining panels and between panel and edge beam. The application of the theory is illustrated by a numerical example.

CHAPTER II

GENERAL MOMENT-REACTION EQUATIONS

2.1. Derivation of Moment Equation. Consider the continuous skew plate of constant flexural stiffness D, subjected to loads normal to the plane of the plate (Figure 2.1). The requirement of compatibility of deformations of adjacent panels m and n is expressed as,

$$\sum \theta_i = 0 \; . \tag{2.1}$$

where:

i is a particular point common to both panels.

Expressing the equation (2.1) in terms of rotation of each panel (Figure 2.1),

$$(\theta_i)_m + (\theta_i)_n = 0 \; . \tag{2.2}$$

where:

$(\theta_i)_m$ and $(\theta_i)_n$ are the rotations at point i of panels m and n respectively.

The slopes can be expressed algeraically as,

$$(\theta)_{im} = \sum^{m} \tau_{ik} \cdot P_k + (F_i)_m M_i + \sum^{m} G_{il} \cdot M_l - \sum^{m} R_j Q_{ij} \tag{2.3}$$

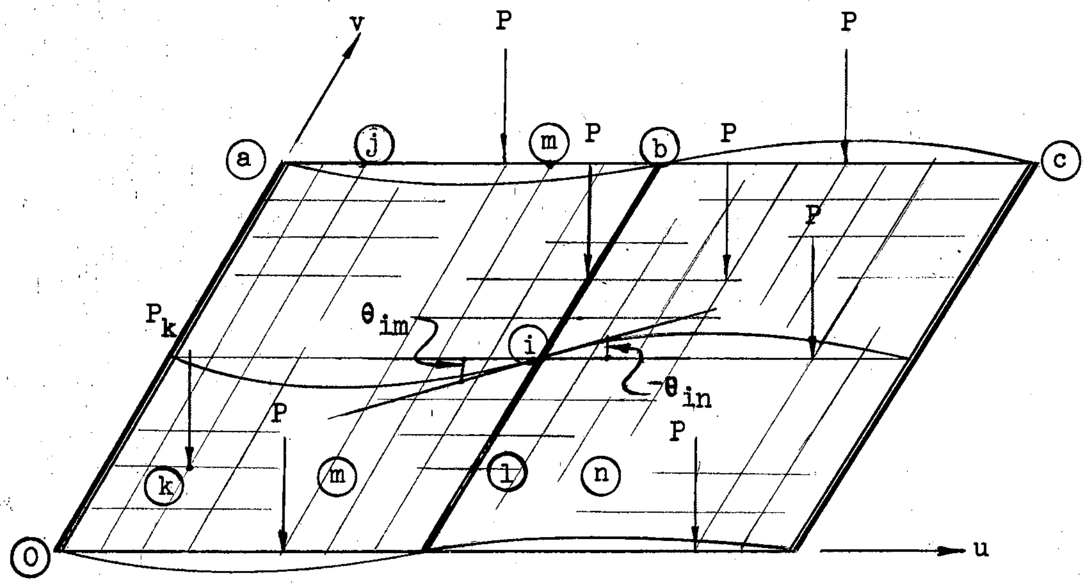

Figure 2.1. Slope Comptability of Adjacent Panels

$$(\theta)_{in} = \sum^{n} \tau_{ik} P_k + (F_i)_n M_i + \sum^{n} G_{il} \cdot M_l - \sum^{n} R_j Q_{ij} \qquad (2.4)$$

where:

i is a particular point common to both panels.

j is a particular point on the free edge of a panel.

k is any interior point of panels.

l is any point, other than i, on the simple support.

m is any point, other than j, on the free edge of a panel.

τ_{ik} is the angular load function (the edge slope at point i due to a unit load at point k of the basic structure.

$(F_i)_{m,n}$ is the angular flexibility (the edge slope at point i, of a basic plate m or n respectively, due to a unit moment at i).

G_{ij} is the angular carry-over (the edge slope at point i due to a unit moment at point j of a simply supported plate).

Q_{ij} is the angular-displacement carry-over (the edge slope at point i, due to a unit shear at point j on free edge).

P_k is any load applied at k.

R_j is any shear at j.

M_i is the bending moment at i.

M_j is the bending moment at j.

From expressions (2.2), (2.3) and (2.4), the general moment equation follows

$$\sum^{m,n} \tau_{ik} \cdot P_k + M_i \cdot \sum^{m,n} (F_i) + \sum^{m,n} G_{il} M_l - \sum^{m,n} R_j Q_{ij} = 0 \qquad (2.5)$$

2.2. Derivation of Reaction Equation. Let the shear between the edge beam and the plate at point j be denoted by R_j. The condition for the geometric compatibility between the beam and the plate is:

$$W_j^P - W_j^B = 0 \qquad (2.6)$$

where:

W_j^P is the deflection at point j of the plate due to loads, edge moments and shears from the beam.

W_j^B is the deflection at point j of the beam due to shears from the plate.

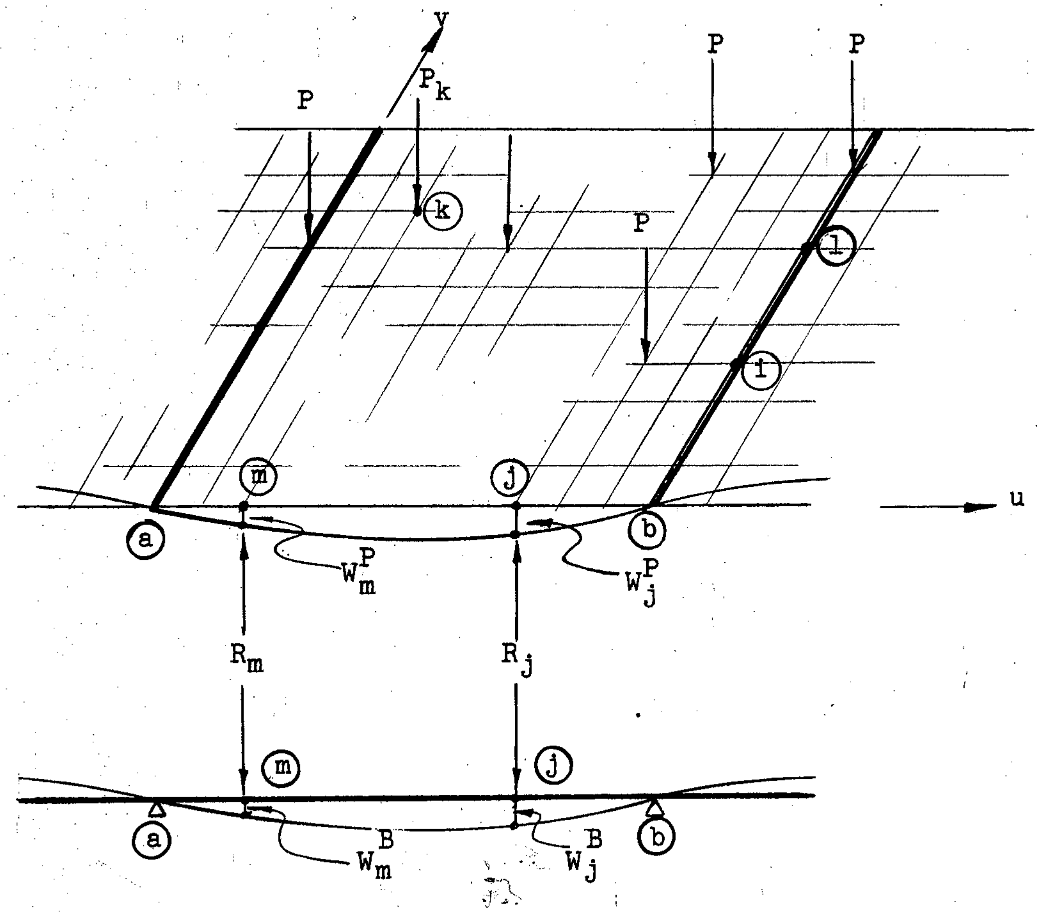

Figure 2.2. Deflection Compatibility of the Beam and the Plate

These deflections can be expressed as,

$$W_j^P = \sum P_k \delta^P_{jk} + \sum M_l Q_{jl} + M_i Q_{ji} - \sum R_m H^P_{jm} - R_j D^P_{jj} \tag{2.7}$$

and,

$$W_j^B = \sum R_m H^B_{jm} + R_j D^B_{jj} + M_a Q^B_{ja} + M_b Q^B_{jb} + \delta^B_j \tag{2.8}$$

expression (2.6), after substitution of (2.7) and (2.8), yields

$$M_i Q^P_{ji} + \sum M_l Q^P_{jl} - M_a Q^B_{ja} - M_b Q^B_{jb} + \sum P_k \delta_{jk} - R_j D_{jj} \tag{2.9}$$

$$- \delta^B_j - R_m H_{jm} = 0$$

where:

δ_{jk} is the displacement load function (the deflection at point j on the free edge of the plate due to a unit load k).

Q_{jl} is the displacement-angular carry-over of the plate (the deflection at point j on the free edge of the plate due to a unit moment applied at l).

D^P_{jj} is the displacement flexibility of the plate (the deflection at point j on the free edge due to a unit load at that point).

H^P_{mj} is the displacement carry-over of the plate (the deflection at point m on the free edge due to a unit load at point j).

δ^B_j is the deflection at point j of the beam (due to the weight of the beam).

Q^B_{ja} is the displacement-angular carry-over of the beam (the deflection at point j on the beam due to a unit moment at a).

D^B_{jj} is the displacement flexibility of the beam (the deflection at point j of the beam due to a unit load at point j).

H^B_{mj} is the displacement carry-over of the beam (the deflection at point m of the beam due to a unit load at point j).

$$D_{jj} = (D^P_{jj} + D^B_{jj})$$

$$H_{mj} = (H^P_{mj} + H^B_{mj})$$

2.3. Matrix Formulation. In the case when it is necessary to use a relatively fine network and the number of panels is large, solution by electronic computer becomes essential. In such a case, it is convenient to have a general matrix formulation of the moment and reaction equations. The general matrix equations for a continuous plate problem having total of $L + m$ unknown moments and reactions is given by Equation (2.10)

$$
\begin{bmatrix}
\Sigma F_1 & G_{1,2} & G_{1,3} & - & - & G_{1,L} & Q_{1,j} & - & - & Q_{1,m} \\
G_{2,1} & \Sigma F_2 & G_{2,3} & - & - & G_{2,L} & G_{2,j} & - & - & Q_{2,m} \\
G_{3,1} & G_{3,2} & \Sigma F_3 & - & - & G_{3,L} & Q_{3,j} & - & - & Q_{3,m} \\
- & - & - & - & - & - & - & - & - & - \\
- & - & - & - & - & - & - & - & - & - \\
- & - & - & - & - & - & - & - & - & - \\
G_{L,1} & G_{L,2} & G_{L,3} & - & - & \Sigma F_L & Q_{L,j} & - & - & Q_{L,m} \\
Q^P_{j,1} & Q^P_{j,2} & Q^P_{j,3} & - & - & Q^P_{j,L} & D_{jj} & - & - & H_{jm} \\
- & - & - & - & - & - & - & - & - & - \\
- & - & - & - & - & - & - & - & - & - \\
Q^P_{m,1} & Q^P_{m,2} & Q^P_{m,3} & - & - & Q^P_{m,L} & H_{mj} & - & - & D_{mm}
\end{bmatrix}
\begin{bmatrix}
M_1 \\ M_2 \\ M_3 \\ - \\ - \\ - \\ M_L \\ -R_j \\ - \\ - \\ -R_m
\end{bmatrix}
+
\begin{bmatrix}
\tau_{1k}P_k \\ \tau_{2k}P_k \\ \tau_{3k}P_k \\ - \\ - \\ - \\ \tau_{Lk}P_k \\ \delta_{jk} \\ - \\ - \\ \delta_{mk}
\end{bmatrix}
= 0 \qquad (2.10)
$$

The General Moment-Reaction Matrix Equation

CHAPTER III

ANGULAR AND LOAD FUNCTIONS FOR PLATE

3.1. Angular Load Function. Consider a skew plate simply supported on two parallel edges, the other two edges free, to be acted upon by a load $P = 1$ at point k (Figure 3.1). The slope at i due to a unit load at k is the angular load function, τ_{ik}. It can be expressed mathematically as,

$$\tau_{ik} = \frac{W_{i+1}}{\Delta u} \tag{3.1}$$

where:

W_{i+1} is the deflection of the point $i+1$.

Δu is the small distance in u direction.

This deflection due to a unit load at k is

$$W_{i+1,k} = \frac{1}{D} \Delta u \cdot \Delta v \cdot \sin\alpha \cdot \eta_{i+1,k} \tag{3.2}$$

where:

$\eta_{i+1,k}$ is the influence coefficient for deflection at $i+1$ due a unit load at k.

Using expressions (3.1) and (3.2), the final equation for the angular load function becomes

$$\tau_{ik} = \frac{\Delta v \cdot \sin\alpha}{D} \cdot \eta_{i+1,k} \tag{3.3}$$

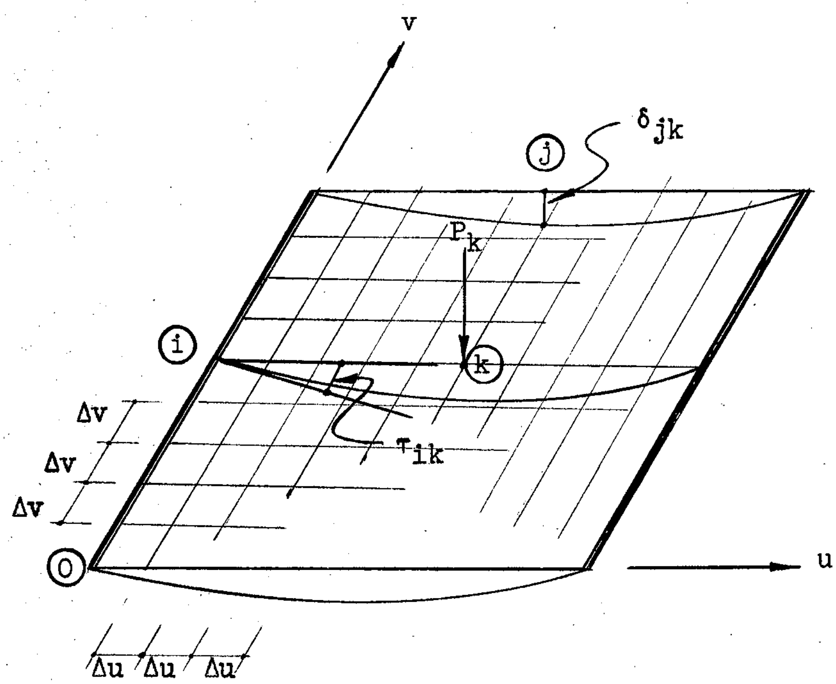

Figure 3.1. Angular and Displacement Load Functions in a Basic Plate

3.2. <u>Angular Flexibility</u>. Consider the basic skew panel to be acted upon by a moment $M_i = 1$ at point i in the u direction (Figure 3.2). The slope at i due to a unit moment at i is defined as the angular flexibility, F_{ii}.

Using the moment equivalence shown in (Figure 3.3), the angular flexibility can be approximated as,

$$F_{ii} = \frac{W_{i+1,\ i+1}}{\Delta u} \tag{3.4}$$

where:

$W_{i+1,\ i+1}$ is the deflection at point $i+1$ due to a load intensity of $\frac{1}{\Delta u}$ at point $i+1$.

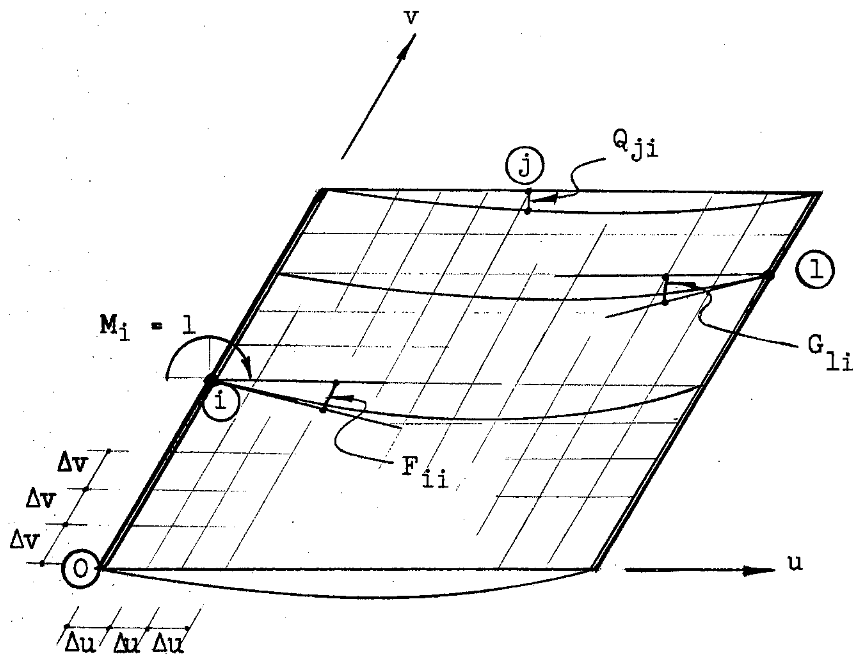

Figure 3.2. Angular Flexibility and Carry-Over in a Basic Plate

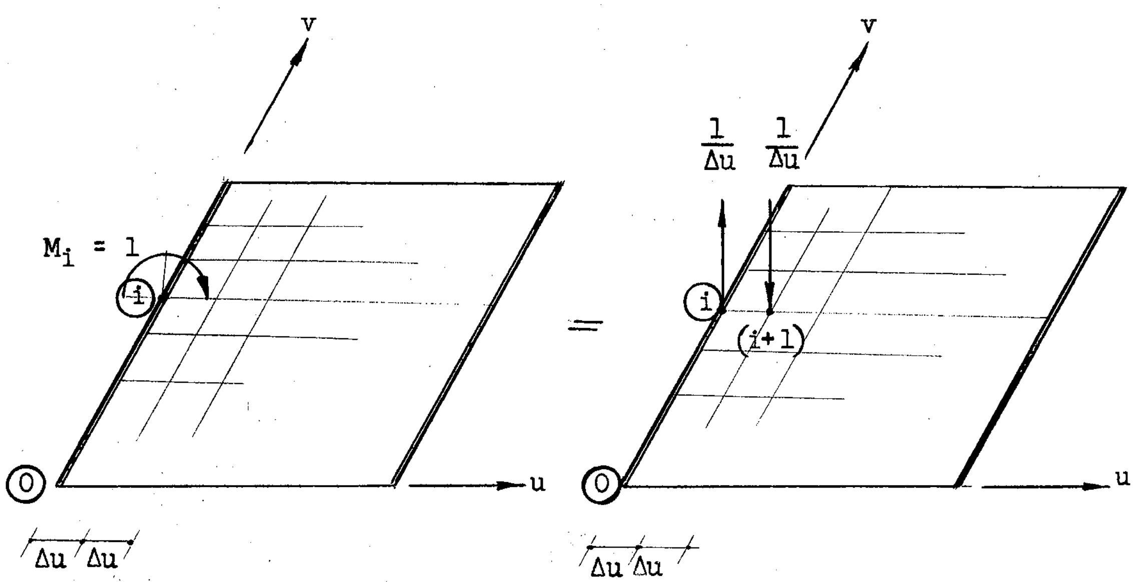

Figure 3.3. Moment Equivalence in a Basic Plate

$$W_{i+1,i+1} = \frac{\frac{1}{\Delta u}}{D} \cdot \Delta u \cdot \Delta v \cdot \sin\alpha \cdot \eta_{i+1,i+1} \tag{3.5}$$

substitution of equation (3.5) into (3.4) gives

$$F_{ii} = \frac{\sin\alpha}{t \cdot D} \cdot \eta_{i+1,\ i+1} \tag{3.6}$$

Equation (3.6) is the final expression for angular flexibility in terms of deflection influence coefficient.

3.3. <u>Angular Carry-Over</u>. The angular carry-over, G_{ji}, can be stated as the slope at j due to unit moment $M_i = 1$ applied at i (Figure 3.2).

$$G_{ji} = \frac{W_{j-1}}{\Delta u} \tag{3.7}$$

where:

W_{j-1} is the deflection at $j-1$ due to a unit moment $M_i = 1$ at point i.

Again using the moment equivalence of Figure 3.3,

$$W_{j-1} = \frac{\frac{1}{\Delta u}}{D} \Delta u \cdot \Delta v \cdot \sin\alpha \cdot \eta_{j-1,\ i+1} \tag{3.8}$$

Combining (3.7) with (3.8), the final expression for angular carry-over is

$$G_{ji} = \frac{\sin\alpha}{t \cdot D} \eta_{j-1,i+1} \tag{3.9}$$

3.4. <u>Displacement Load Function</u>. The displacement load function, δ_{jk}, is defined as the deflection of any point j on free edge due to a unit load at point k (Figure 3.1).

$$\delta_{jk} = \frac{\Delta A}{D} \cdot \eta_{jk} \tag{3.10}$$

Equation (3.10) is the final expression for displacement load function.

3.5. Displacement Flexibility. Considering the skew plate of Figure (3.4) the displacement flexibility, $-D_{jj}$, is defined as the deflection of point j of a basic panel due to a unit vertical shear $R_j = 1$ from the edge beam, j being a point on the free edge.

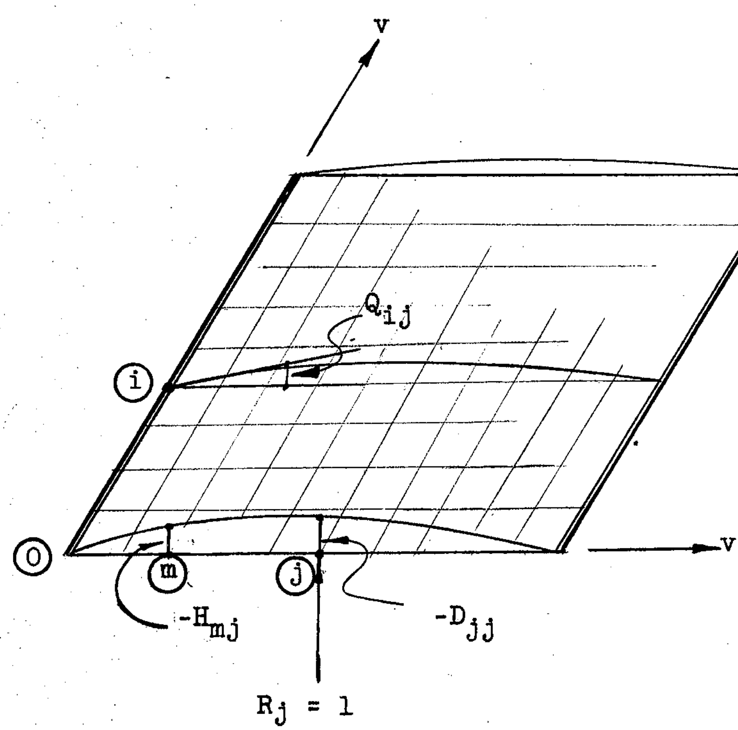

Figure 3.4. Displacement Flexibility and Carry-Over in a Basic Plate

The final form of displacement flexibility is

$$D_{jj} = \frac{\Delta A}{D} \eta_{jj} \tag{3.11}$$

3.6. Displacement Carry-Over. The displacement carry-over, $-H_{mj}$, is defined as the deflection of point m on the free edge due to a unit vertical shear $R_j = 1$ (Figure 3.4). It can be expressed mathematically as,

$$H_{mj} = \frac{\Delta A}{D} \eta_{mj} \tag{3.12}$$

3.7. Displacement-Angular Carry-Over. The displacement-angular carry-over, Q_{ji}, is defined as the deflection of any point j on free edge due to a unit moment $M_i = 1$ applied at i (Figure 3.2). The final form of displacement-angular carry-over is

$$Q_{ji} = \frac{\Delta v \cdot \sin\alpha}{D} \eta_{ji} \quad . \tag{3.13}$$

CHAPTER IV

ANGULAR AND LOAD FUNCTIONS FOR BEAM

4.1. Angular Flexibility and Carry-Over. Consider the beam of Figure (4.2), simply supported at both ends and acted upon by a unit moment $M_a = 1$ at end a.

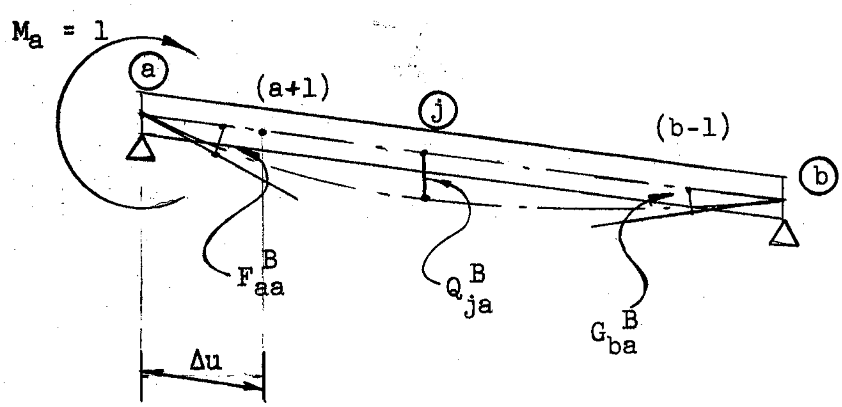

Figure 4.1. Angular Flexibility and Carry-Over for a Basic Beam

By definition the angular flexibility F^B_{aa} of the beam is the slope at a due to a unit moment at a. Using the moment equivalence of Figure (4.2), the angular flexibility can be expressed as,

$$F^B_{aa} = \frac{D^B_{a+1\ ,a+1}}{\Delta u} \cdot \frac{1}{\Delta u} \tag{4.1}$$

where:

$$D^B_{a+1\ ,a+1} = \frac{(\Delta u)^3}{B}\ \eta^B_{a+1,\ a+1} \tag{4.2}$$

The final form of the angular flexibility is

$$F^B_{aa} = \frac{\Delta u}{B}\ \eta^B_{a+1,\ a+1} \tag{4.3}$$

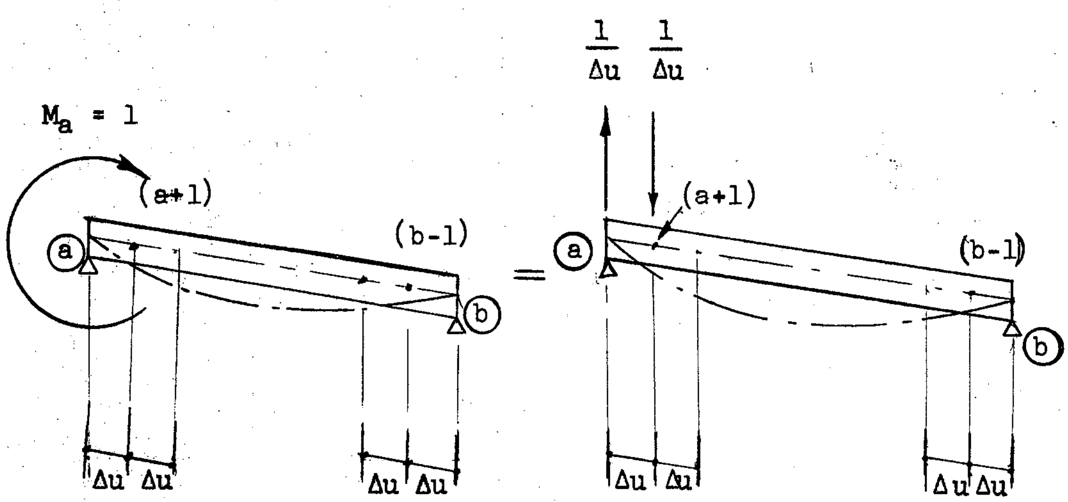

Figure 4.2. The Moment Equivalence for the Edge Beam

G^B_{ba}, the angular carry-over of the basic beam, is the slope at b due to a unit moment applied at a. From Figures (4.1) and (4.2)

$$G^B_{ba} = \frac{H^B_{b-1,a+1}}{\Delta u} \cdot \frac{1}{\Delta u} \tag{4.4}$$

where:

$$H^B_{b-1,a+1} = \frac{(\Delta u)^3}{B} \eta^B_{b-1,a+1} \tag{4.5}$$

Combining equations (4.4) and (4.5)

$$G^B_{ba} = \frac{\Delta u}{B} \eta^B_{b-1,a+1} \tag{4.6}$$

4.2. Displacement Angular Function. By definition the displacement angular function, Q^B_{ja}, is the deflection of point j due to a unit moment applied at end a (Figure 4.1).

$$Q^B_{ja} = \frac{(\Delta u)^2}{B} \eta^B_{j,a+1} \tag{4.7}$$

Equation (4.7) is the final expression for the displacement angular function.

4.3. Displacement Flexibility and Carry-Over for Beam. Consider the edge beam of Figure (4.3), simply supported at both ends and subjected to a unit vertical load $R_j = 1$ at point j. The flexural stiffness EI of the beam is denoted by B and is assumed to be constant. The displacement flexibility of the beam is denoted by D^B_{jj} and is defined as the deflection of point j due to a unit vertical load $R_j = 1$ at point j.

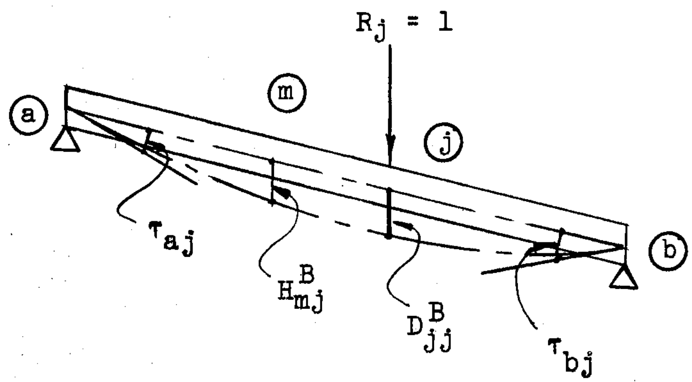

Figure 4.3. Displacement Flexibility and Displacement Carry-Over for a Beam

The displacement flexibility in terms of the deflection influence coefficients can be mathematically expressed as,

$$D_{jj}^{B} = \frac{(\Delta u)^3}{B} \eta_{jj}^{B} \tag{4.8}$$

The displacement carry-over of the beam, H_{mj}^{B}, is defined as the deflection of point m due to a unit vertical load $R_j = 1$ at point j.

$$H_{mj}^{B} = \frac{(\Delta u)^3}{B} \eta_{mj}^{B} \tag{4.9}$$

CHAPTER V

GENERAL DEFLECTION EQUATIONS

5.1. Derivation of General Deflection Equation. A skew plate panel, subjected to loads normal to the plane of the plate as shown in Figure 5.1, is isolated from a series of skew plates continuous in one direction. The flexural rigidity, D, of the plate is assumed to be constant.

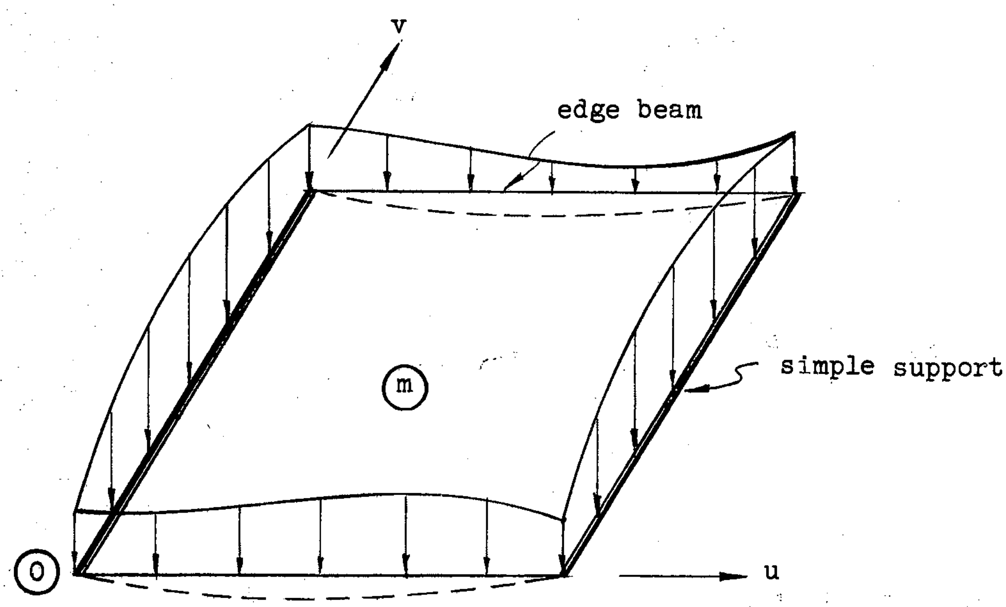

Figure 5.1. General Isolated Panel

From the ordinary theory of thin plates the deflected surface of the plate must satisfy the differential equation

$$D \nabla^4 w = p \qquad (5.1)$$

where:

w is the deflection of the plate.

p is the intensity of the load.

For a plate of thickness h, the flexural rigidity D can be expressed as

$$D = \frac{Eh^3}{12(1-\mu)^2} \qquad (5.2)$$

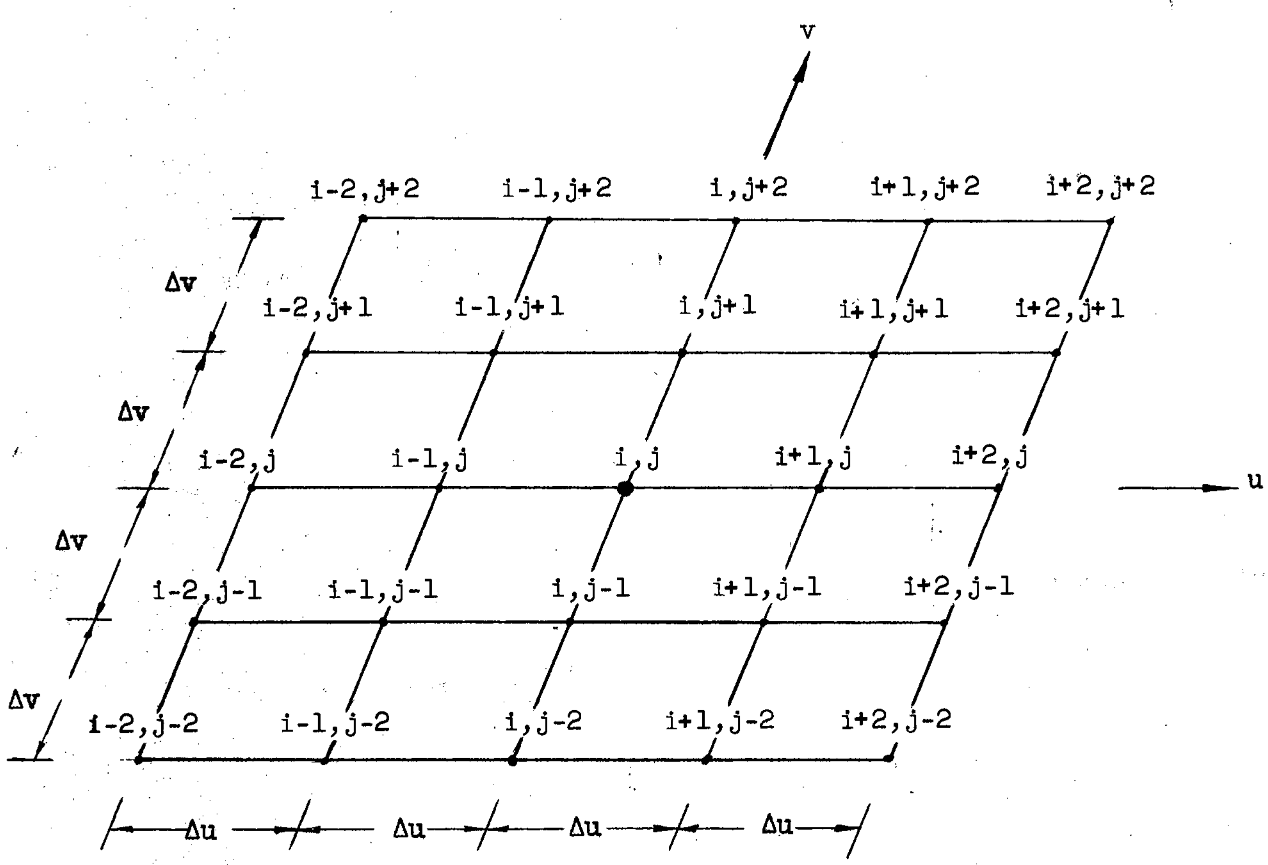

Figure 5.2. Skew Network

Expression (5.1) can be resolved into two parts as

$$\nabla^2 w = -\frac{M}{D} \tag{5.3}$$

and

$$\nabla^2 M = -p \tag{5.4}$$

In equation (5.3) ∇^2, the Laplacian operator (8) in skew co-ordinates, is expressed by

$$\nabla^2 = \frac{1}{\sin^2\alpha}\cdot\frac{\partial^2}{\partial u^2} + \frac{1}{\sin^2\alpha}\cdot\frac{\partial^2}{\partial v^2} - \frac{2\cdot\cos\alpha}{\sin^2\alpha}\cdot\frac{\partial^2}{\partial u\cdot\partial v} \tag{5.5}$$

Equation (5.4) can therefore be written as

$$-p = \frac{1}{\sin^2\alpha}\cdot\frac{\partial^2 M}{\partial u^2} + \frac{1}{\sin^2\alpha}\cdot\frac{\partial^2 M}{\partial v^2} - \frac{2\cos\alpha}{\sin^2\alpha}\cdot\frac{\partial^2 M}{\partial u\cdot\partial v} \tag{5.6}$$

Expressing the partial derivatives in Equation (5.6) by finite differences, the resulting equation for any interior point ij as defined by the skew network of Figure 5.2, is given by an equation (5.7).

$$p_{ij}(\lambda\Delta A) = \begin{array}{ccccc} -c & — & -b & — & c \\ -a & — & 1 & — & -a \\ c & — & -b & — & -c \end{array} \quad (M) \tag{5.7a}$$

$$\frac{M_{ij}}{D}(\lambda\Delta A) = \begin{array}{ccccc} -c & — & -b & — & c \\ / & & / & & / \\ -a & — & 1 & — & -a \\ / & & / & & / \\ c & — & -b & — & -c \end{array} \quad (W) \qquad (5.7b)$$

where:

$$t = \frac{\Delta u}{\Delta v}$$

$$a = \frac{1}{2(1+t^2)}$$

$$b = \frac{t}{2(1+t^2)}$$

$$c = \frac{t\cdot\cos\alpha}{4(1+t^2)}$$

$$\Delta A = \Delta u\cdot\Delta v\cdot\sin\alpha$$

By a simple interchange of subscripts the expressions for

$M_{i+1,j}$ $M_{i-1,j}$ $M_{i+1,j+1}$ $M_{i+1,j-1}$ $M_{i-1,j-1}$

$M_{i-1,j+1}$ $M_{i,j+1}$ and $M_{i,j-1}$

may be written corresponding to Equation (5.7b):

$$M_{i,j+1}\frac{\lambda\Delta A}{D} = W_{i,j+1} - a(W_{i-1,j+1} + W_{i+1,j+1}) - b(W_{i,j+2} + W_{i,j}) - c(W_{i-1,j+2} + W_{i+1,j} - W_{i+1,j+2} - W_{i-1,j})$$

$$M_{i,j-1}\frac{\lambda\Delta A}{D} = W_{i,j-1} - a(W_{i-1,j-1} + W_{i+1,j-1}) - b(W_{i,j} + W_{i,j-2}) - c(W_{i-1,j} + W_{i+1,j-2} - W_{i+1,j} - W_{i-1,j-2})$$

$$M_{i+1,j+1}\frac{\lambda\Delta A}{D} = W_{i+1,j+1} - a(W_{i,j+1} + W_{i+2,j+1}) - b(W_{i+1,j+2} + W_{i+1,j}) - c(W_{i,j+2} + W_{i+2,j} - W_{i+2,j+2} - W_{i,j})$$

$$M_{i+1,j}\frac{\lambda\Delta A}{D} = W_{i+1,j} - a(W_{i,j} + W_{i+2,j}) - b(W_{i+1,j+1} + W_{i+1,j-1}) - c(W_{i,j+1} + W_{i+2,j-1} - W_{i+2,j+1} - W_{i,j-1})$$

$$M_{i+1,j-1}\frac{\lambda\Delta A}{D} = W_{i+1,j-1} - a(W_{i,j-1} + W_{i+2,j-1}) - b(W_{i+1,j} + W_{i+1,j-2}) - c(W_{i,j} + W_{i+2,j-2} - W_{i+2,j} - W_{i,j-2})$$

$$M_{i-1,j+1}\frac{\lambda\Delta A}{D} = W_{i-1,j+1} - a(W_{i-2,j+1} + W_{i,j+1}) - b(W_{i-1,j+2} + W_{i-1,j}) - c(W_{i-2,j+2} + W_{i,j} - W_{i,j+2} - W_{i-2,j})$$

$$M_{i-1,j}\frac{\lambda\Delta A}{D} = W_{i-1,j} - a(W_{i-2,j} + W_{i,j}) - b(W_{i-1,j+1} + W_{i-1,j-1}) - c(W_{i-2,j+1} + W_{i,j-1} - W_{i,j+1} - W_{i-2,j-1})$$

$$M_{i-1,j-1}\frac{\lambda\Delta A}{D} = W_{i-1,j-1} - a(W_{i-2,j-1} + W_{i,j-1}) - b(W_{i-1,j} + W_{i-1,j-2}) - c(W_{i-2,j} + W_{i,j-2} - W_{i,j} - W_{i-2,j-2}) \tag{5.8}$$

Substitution of Equations (5.7b) and (5.8) into Equation (5.7a) yields after simplication,

$$\frac{P_{ij}}{D}(\lambda \cdot \Delta A)^2 = \begin{array}{ccccccccc} c^2 & - & 2bc & - & (b^2-2c^2) & - & -2bc & - & c^2 \\ | & & | & & | & & | & & | \\ 2ac & - & (2ab-2c) & - & -2b & - & (2ab+2c) & - & -2ac \\ | & & | & & | & & | & & | \\ (a^2-2c^2) & - & -2a & - & \begin{array}{c}(1+2a^2+2b\ \ )\\ 4c\end{array} & - & -2a & - & (a^2-2c^2) \\ | & & | & & | & & | & & | \\ -2ac & - & (2ab+2c) & - & -2b & - & (2ab-2c) & - & -2ac \\ | & & | & & | & & | & & | \\ c^2 & - & -2bc & - & (b^2-2c^2) & - & 2bc & - & c^2 \end{array} \quad (w)$$

(5.9)

An expression similar to Equation (5.9) may be written for any interior point of the network if it is surrounded by a sufficient number of points on all the sides. For points on or adjacent to the boundary Equation (5.9) can be modified by using appropriate boundary conditions.

Boundary conditions at the simple support can be expressed mathematically as,

$$(w)_{edge} = 0$$

$$(\nabla^2 w)edge = (M)edge = 0 \qquad (5.10)$$

which can be derived from the conditions that there shall be no deflection on the line of simple support and there shall be no moment normal to the simple support, on the line of the simple support.

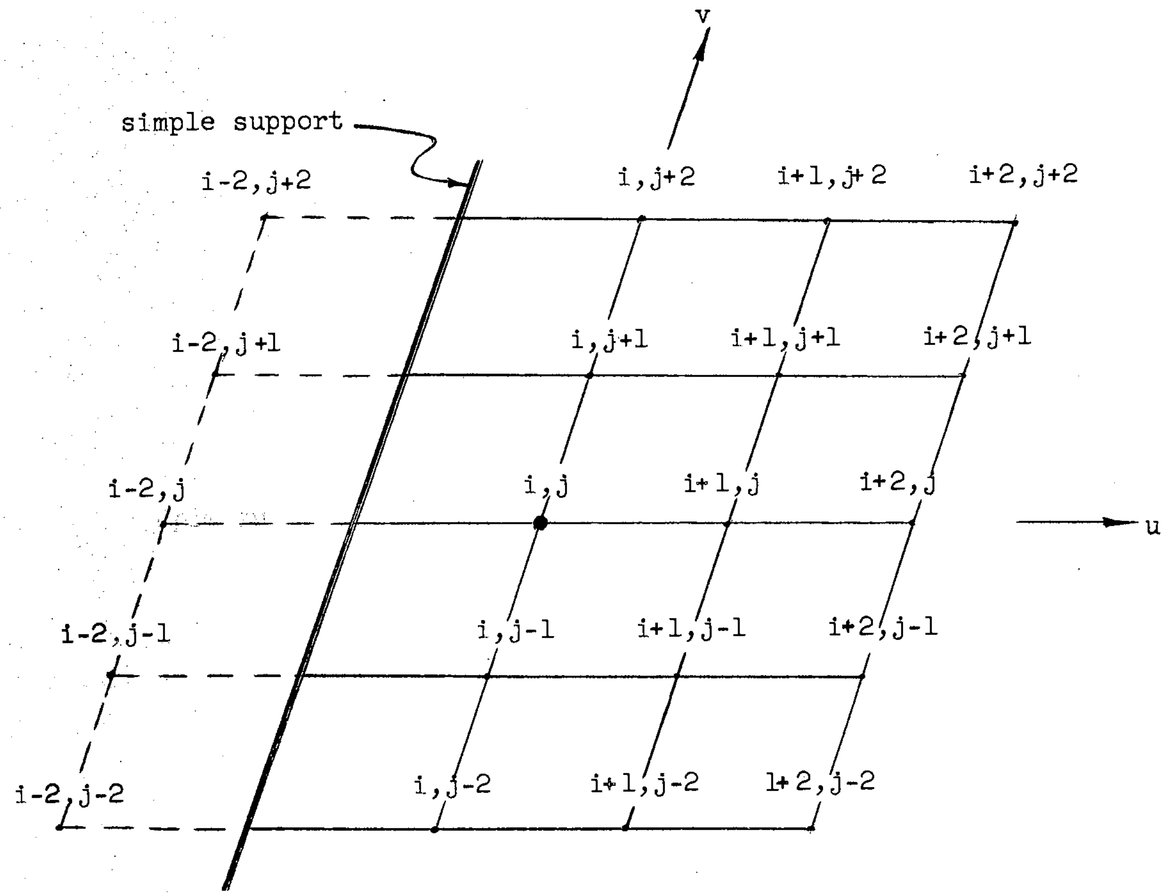

Figure 5.3. Point ij on the First Interior Line Parallel to Simple Support

For point ij on the first interior line parallel to a simply supported edge as shown in Figure (5.3), the boundary conditions given by Equation (5.10) may be written as

$$W_{i-1,j} = W_{i-1,j+1} = W_{i-1,j+2} = W_{i-1,j-1} = W_{i-1,j-2} = 0$$

and

$$M_{i-1,j+1} = M_{i-1,j} = M_{i-1,j-1} = 0 \qquad (5.11)$$

The remaining values of M from Equations (5.7b) and (5.8), when substituted in Equation (5.3), yield, after proper modifications obtained by Equation (5.11),

$$\frac{p_{ij}}{D}(\lambda\cdot\Delta A)^2 = \begin{array}{ccccc} \boxed{(b^2-c^2)} & - & \boxed{-2bc} & - & \boxed{c^2} \\ \boxed{-2b} & - & \boxed{2(ab+c)} & - & \boxed{-2ac} \\ \boxed{(1+a^2+2b^2+2c^2)} & - & \boxed{-2a} & - & \boxed{(a^2-2c^2)} \\ \boxed{-2b} & - & \boxed{2(ab-c)} & - & \boxed{2ac} \\ \boxed{(b^2-c^2)} & - & \boxed{2bc} & - & \boxed{c^2} \end{array} \tag{5.12}$$

In similar manner, an equation for any point ij on the first interior line parallel to the opposite simply supported edge may be obtained.

Consider now a point ij on the first interior line parallel to the free edge as shown in Figure 5.4. Boundary conditions on the free edge are

$$(M_y)_{edge} = -D\left[\frac{\partial^2 w}{\partial y^2} + \mu\frac{\partial^2 w}{\partial x^2}\right] = 0 \tag{5.13}$$

and

$$(R_y)_{edge} = -D\left[\frac{\partial}{\partial y}(\nabla^2 w) + (1-\mu)\frac{\partial}{\partial y}\circ\frac{\partial^2 w}{\partial u^2}\right] = 0 \tag{5.14}$$

Rewriting Equation (5.13), utilizing skew coordinates,

$$(\nabla^2 w)_{edge} = (1-\mu)\left(\frac{\partial^2 w}{\partial u^2}\right)_{edge} = -\frac{(M)edge}{D} \tag{5.15}$$

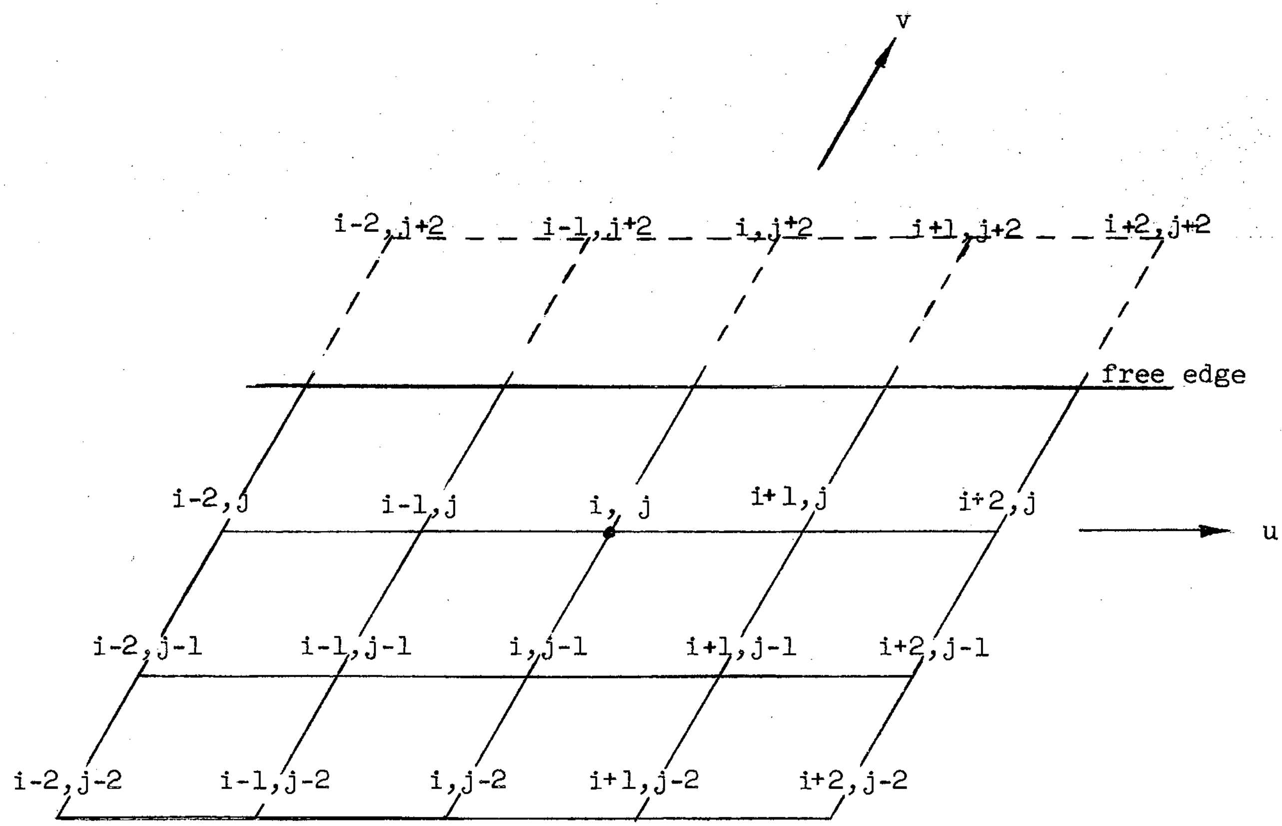

Figure 5.4. Point ij on Line Adjacent to Free edge

Boundary condition (5.15), when applied to points $(i, j+1)$, $(i-1, j+1)$, and $(i+1, j+1)$ gives

$$\frac{1}{D} M_{i,j+1} = -\frac{(1-\mu)}{\Delta u^2} (W_{i-1,j+1} - 2W_{i,j+1} + W_{i+1,j+1})$$

$$\frac{1}{D} M_{i+1,j+1} = -\frac{(1-\mu)}{\Delta u^2} (W_{i,j+1} - 2W_{i+1,j+1} + W_{i+2,j+1}) \qquad (5.16)$$

$$\frac{1}{D} M_{i-1,j+1} = -\frac{(1-\mu)}{\Delta u^2} (W_{i-2,j+1} - 2W_{i-1,j+1} + W_{i,j+1})$$

Denoting,

$$d = \frac{(1-\mu)}{\Delta u^2} \cdot \lambda \cdot \Delta A = \frac{(1-\mu)}{t} \cdot \lambda \cdot \sin\alpha \qquad (5.17)$$

Equation (5.16) becomes

$$M_{i,j+1} \frac{\lambda \cdot \Delta A}{D} = -d \, (W_{i-1,j+1} - 2W_{i,j+1} + W_{i+1,j+1})$$

$$M_{i+1,j+1} \frac{\lambda \cdot \Delta A}{D} = -d \, (W_{i,j+1} - 2W_{i+1,j+1} + W_{i+2,j+1}) \qquad (5.18)$$

$$M_{i-1,j+1} \frac{\lambda \cdot \Delta A}{D} = -d \, (W_{i-2,j+1} - 2W_{i-1,j+1} + W_{i,j+1})$$

These values of M from Equations (5.18) together with remaining values of M from (5.7b) and (5.8), when substituted in Equation (5.7a), finally yield

$$p_{ij}\frac{(\lambda \Delta A)^2}{D} = \begin{array}{ccccc} c(a+d) & \begin{array}{c}-c(1+2d)\\+b(a+d)\end{array} & -b(1+2d) & \begin{array}{c}c(1+2d)\\+b(a+d)\end{array} & -c(a+d) \\ (a^2-c^2) & -2a & (1+2a^2+b^2+2c^2) & -2a & (a^2-c^2) \\ -2ac & 2(ab+c) & -2b & 2(ab-c) & 2ac \\ c^2 & -2bc & (b^2-2c^2) & 2bc & c^2 \end{array} \quad (w)$$

(5.19)

When point ij is an interior point near a sharp corner as shown in Figure 5.5, the boundary conditions are,

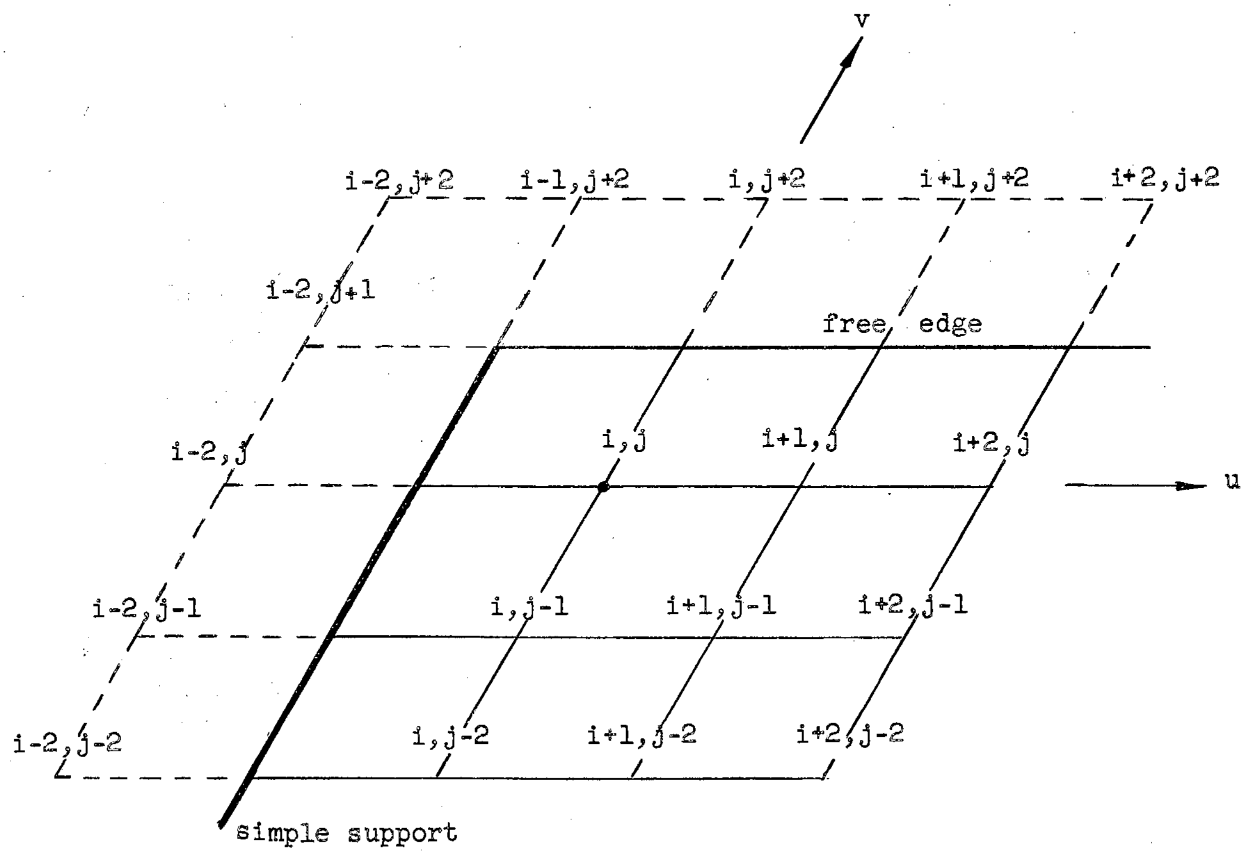

Figure 5.5. Point ij Near Corner

$$W_{i-1,j+1} = W_{i-1,j} = W_{i-1,j-1} = W_{i-1,j-2} = 0 \tag{5.20}$$

$$M_{i-1,j+1} = M_{i-1,j} = M_{i-1,j-1} = 0$$

$$M_{i,j+1}\frac{\lambda\Delta A}{D} = -d(W_{i+1,j+1} - 2W_{i,j+1}) \tag{5.21}$$

$$M_{i+1,j+1}\frac{\lambda\Delta A}{D} = -d(W_{i,j+1} \;-\; 2W_{i+1,j+1} \quad W_{i+2,j+1})$$

Substitution of values for M and W from Equations (5.7b), (5.8), (5.20) and (5.21) into Equation (5.7a) results in the following equation for point ij of Figure (5.5):

$$p_{ij}\,\frac{(\lambda\Delta A)^2}{D} \;=\; \begin{array}{ccccc}
\boxed{\begin{array}{c} c(a-d) \\ -b(1+2d) \end{array}} & — & \boxed{\begin{array}{c} b(a+d) \\ +c(1+2d) \end{array}} & — & \boxed{+c(a+d)} \\
/ & & / & & / \\
\boxed{(1+a^2+b^2+c^2)} & — & \boxed{-2a} & — & \boxed{(a^2-c^2)} \\
/ & & / & & / \\
\boxed{-2b} & — & \boxed{2(ab-c)} & — & \boxed{2ac} \\
/ & & / & & / \\
\boxed{(b^2-c^2)} & — & \boxed{2bc} & — & \boxed{c^2}
\end{array} \qquad (w)$$

(5.22)

In the same manner an equation for point ij of Figure (5.6) can be written as,

$$\frac{p_{ij}}{D}(\lambda\Delta A)^2 = \begin{array}{ccc} c(a+d) & \begin{array}{c}+b(a+d)\\ -c(1+2d)\end{array} & \begin{array}{c}-c(a-d)\\ -b(1+2d)\end{array} \\ (a^2-c^2) & -2a & (1+a^2+b^2+c^2) \\ -2ac & 2(ab+c) & -2b \\ c^2 & -2bc & (b^2-c^2) \end{array} \qquad (W) \qquad (5.23)$$

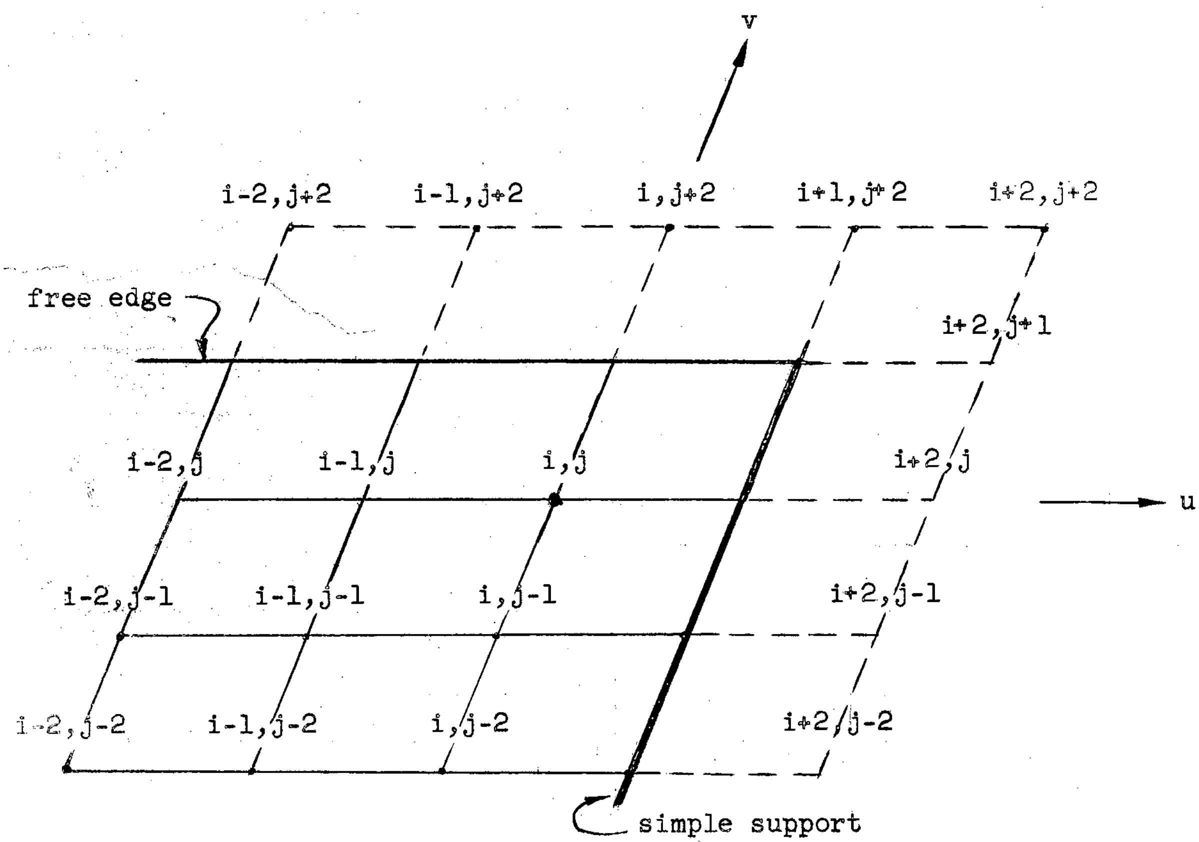

Figure 5.6. Point ij Near Acute Corner

Finally, the deflection equation applicable for any point ij on the free edge of Figure (5.7) must be determined.

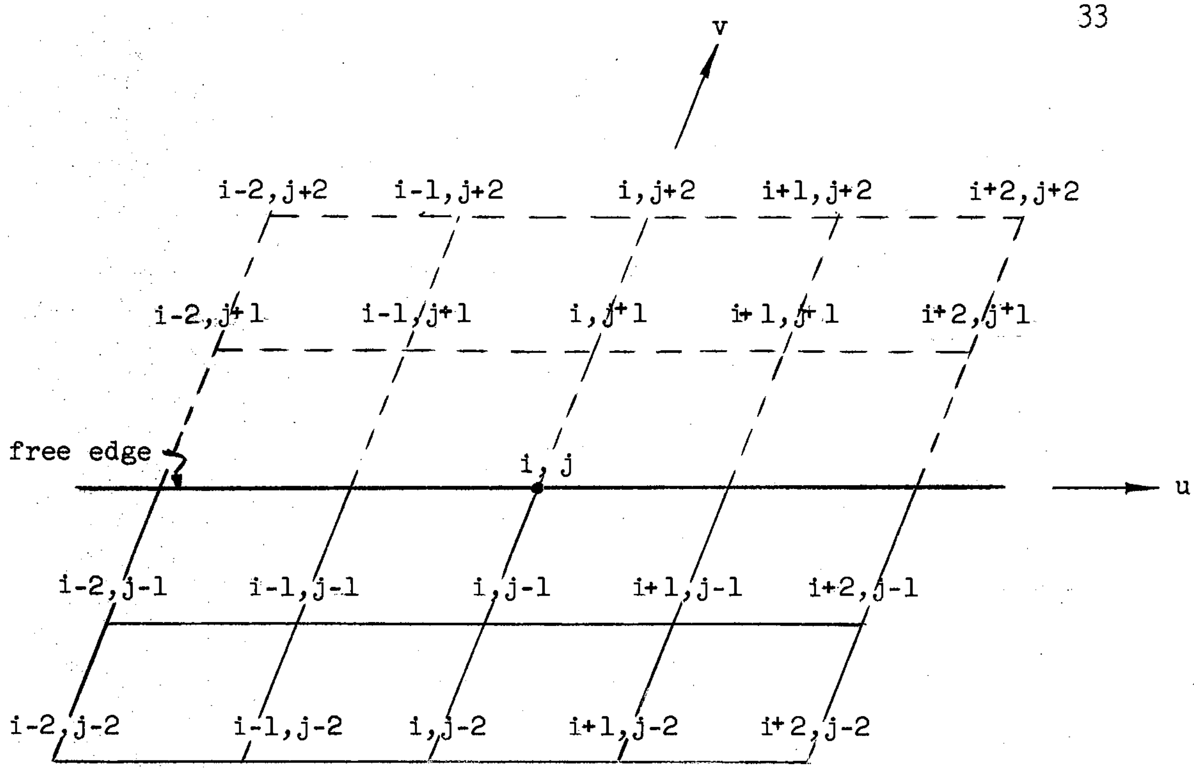

Figure 5.7. Point ij on Free Edge

From Equations (5.14) and (5.15) the boundary conditions can be written as

$$(M)_{edge} = -D(1-\mu)\,\frac{\partial^2 w}{\partial u^2}$$

$$\left[\frac{\partial M}{\partial y}\right]_{edge} = D(1-\mu)\,\frac{\partial}{\partial y}\left(\frac{\partial^2 w}{\partial y^2}\right) \qquad (5.24)$$

Utilizing Jensen's approximation (3) for the normal derivative, and replacing Equations (5.24) by their corresponding finite-difference equations, the deflection equation applicable for a general point ij on the free edge is obtained (Eq. 5.25)

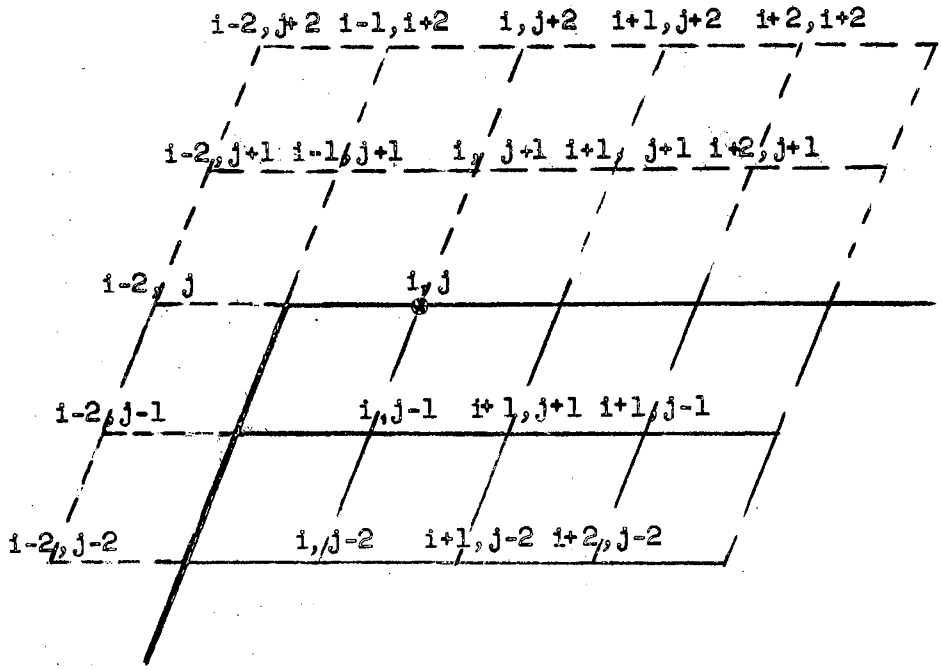

Figure 5.8. Point ij on Free Edge Near Obtuse Corner

$$\frac{P_{ij}(\Delta A)^4}{2N} = \begin{array}{ccccccccc} \boxed{\begin{matrix}-\frac{B^2}{4}-\frac{C^2}{2}\\+AC\end{matrix}} & - & \boxed{\begin{matrix}2C^2-2C\\-4AC\end{matrix}} & - & \boxed{\begin{matrix}1+4C+\frac{B^2}{2}\\-3C+6AC\end{matrix}} & - & \boxed{\begin{matrix}2C^2-2C\\-4AC\end{matrix}} & - & \boxed{\begin{matrix}-\frac{B^2}{4}-\frac{C^2}{2}\\+AC\end{matrix}} \\ | & & | & & | & & | & & | \\ \boxed{-\frac{D}{2}} & - & \boxed{A+B+C+D} & - & \boxed{\begin{matrix}-2-2A\\-2C\end{matrix}} & - & \boxed{A-B+C-D} & - & \boxed{-\frac{D}{2}} \\ | & & | & & | & & | & & | \\ \boxed{\frac{B^2}{4}} & - & \boxed{-B} & - & \boxed{1-\frac{B^2}{2}} & - & \boxed{B} & - & \boxed{\frac{B^2}{4}} \end{array} \quad (W)$$

(5.25)

A deflection equation applicable for point ij on the beam, near the obtuse or acute corner is obtained by modifying Equation (5.25). Equation (5.26) is the final deflection equation for point ij of Figure (5.8).

$$\frac{p_{ij}\,\Delta y^4}{2N} = \begin{array}{ccccc} \boxed{1+4C+\frac{B^2}{4}-\frac{5C^2}{2}+5AC-BC} & — & \boxed{-2C+2C^2-4AC} & — & \boxed{-\frac{B^2}{4}-\frac{C^2}{2}+AC} \\ / & & / & & / \\ \boxed{\frac{D}{2}-AB-2(1+A+C)} & — & \boxed{A+B+C+D} & — & \boxed{-\frac{D}{2}} \\ / & & / & & / \\ \boxed{1-\frac{B^2}{4}} & — & \boxed{-B} & — & \boxed{\frac{B^2}{4}} \end{array} \quad (W) \tag{5.26}$$

The symbols introduced in Equations (5.25) and (5.26) are defined as:

$$\Delta y = \Delta v \cdot \sin\alpha$$

$$K = \frac{\Delta y}{\Delta u}$$

$$A = \left(\frac{K}{\sin\alpha}\right)^2$$

$$B = \frac{K}{\tan\alpha}$$

$$C = (1-\mu)K^2$$

$$D = (A+C)B$$

$$N = \frac{EI}{(1-\mu^2)} = D$$

CHAPTER VI

APPLICATION

The application of the theory developed in the previous chapters is now illustrated by a numerical example. A two-span continuous skew plate-beam system on simple supports is analyzed for uniformly distributed load.

6.1. Procedure of Analysis. The general procedure of analysis is as follows:

1. Evaluate all necessary constants for a single span basic skew panel (carry-over factors a, b, c, etc., of Chapter V).
2. Write deflection equations for each of the points on the network.
3. Solve the set of deflection equations for deflection influence coefficients.
4. Evaluate deflection influence coefficients for beam by statics.
5. Evaluate angular and displacement functions for plate (F, G, τ, δ, Q).
6. Evaluate displacement functions for beam (D, H).
7. Formulate moment-reaction matrix in terms of these functions and redundants.
8. Solve for redundants.

6.2. Deflection Influence Coefficients. Once the general set of deflection equations is formulated for a basic skew panel, the deflection influence coefficients are directly obtained by inverting the coefficients matrix of these equations. To have the capability of solving a wide range of problems it would be necessary to have tables corresponding to a number of length-width ratios and angles of skew. Only one such table is evaluated in this thesis, due to the magnitude of the numerical work involved and the limited computer availability.

A basic 30^o skew panel is considered (Figure 6.1), and is covered by a thirty unit finite difference network. The length-width ratio $\frac{\Delta u}{\Delta v}$ is taken equal to one. Poisson's ratio μ is assumed to be zero.

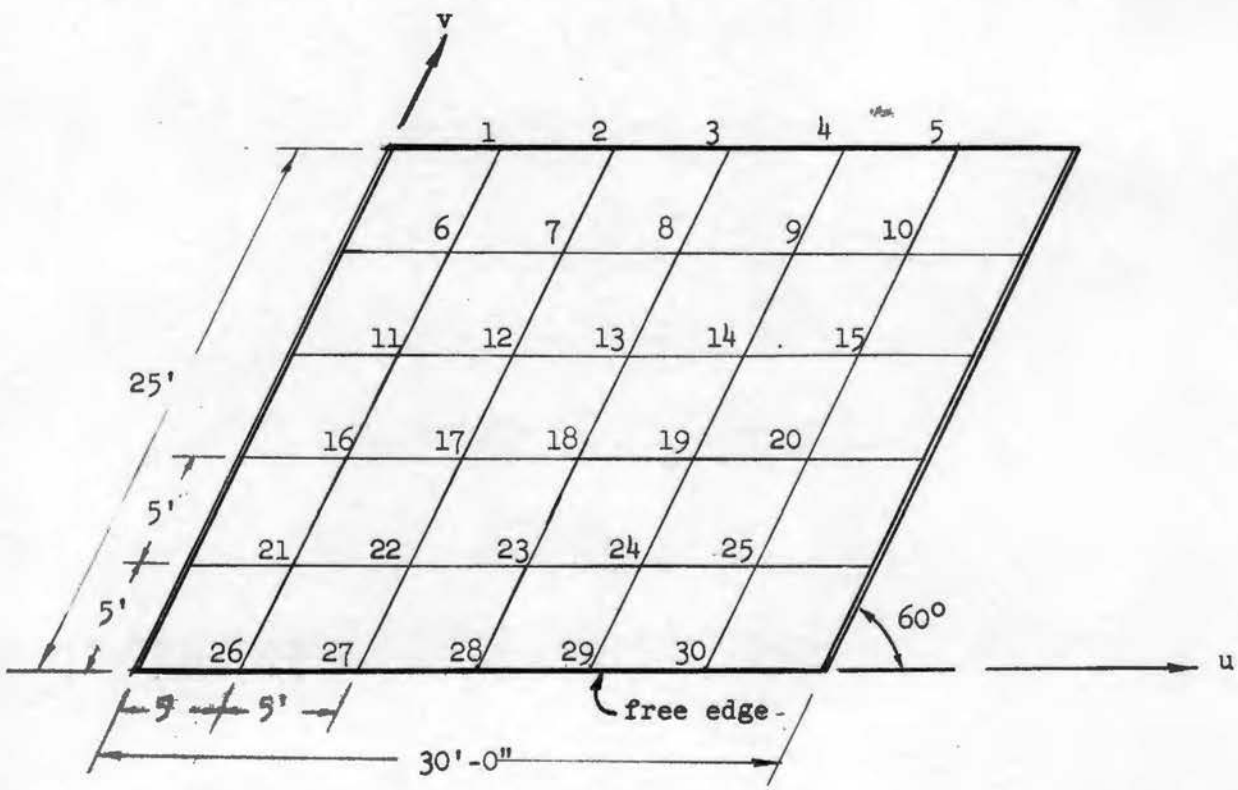

Figure 6.1. Basic Skew Panel.

1. Carry-Over Factors.

$$t = \frac{\Delta u}{\Delta v} = 1.00000$$

$$a = \frac{1}{2(1+t^2)} = .25000$$

$$b = \frac{t}{2(1+t^2)} = .25000$$

$$c = \frac{t \cdot \cos\alpha}{4(1+t^2)} = .06250$$

$$\lambda = \frac{t \cdot \sin\alpha}{2(1+t^2)} = .21651$$

$$d = \frac{(1-\mu)}{t} \cdot \sin\alpha = .18750$$

$$\Delta A = \Delta u \cdot \Delta v \cdot \sin\alpha = 21.6505$$

2. Deflection Equation in Matrix Form. Expressions (5.9), (5.12), (5.19), (5.22), (5.23), (5.27) and (5.28) are used to write deflection equations for points on the network.

Equation (6.1)

$$
\left[\begin{array}{ccccccccccc}
10.74995 & -5.35850 & .64562 & 0.0 & 0.0 & -7.24966 & .49991 & .58326 & 0.0 & 0.0 & 1.24988 \\
-5.35850 & 10.83330 & -5.35850 & .64562 & 0.0 & 4.16624 & -7.33302 & .49991 & .58326 & 0.0 & -.66662 \\
.64562 & -5.35850 & 10.83330 & -5.35850 & .64562 & -.58326 & 4.16624 & -7.33302 & .49991 & .58326 & .08335 \\
0.0 & .64562 & -5.35850 & 10.83330 & -5.35850 & 0.0 & -.58326 & 4.16624 & -7.33302 & .49991 & 0.0 \\
0.0 & 0.0 & .64562 & -5.35850 & 9.41664 & 0.0 & 0.0 & -.58326 & 4.16624 & -7.41638 & 0.0 \\
-7.24966 & 4.16624 & -.58326 & 0.0 & 0.0 & 24.06442 & -11.29414 & 1.24988 & 0.0 & 0.0 & -13.49996 \\
.49991 & -7.33302 & 4.16624 & -.58326 & 0.0 & -13.49996 & 25.47978 & -13.49996 & 1.24988 & 0.0 & 6.74998 \\
.58326 & .49991 & -7.33302 & 4.16624 & -.58326 & 1.24988 & -13.49996 & 25.47978 & -13.49996 & 1.24988 & -.66662 \\
0.0 & .58326 & .49991 & -7.33302 & 4.16624 & 0.0 & 1.24988 & -13.49996 & 25.47978 & -13.49996 & 0.0 \\
0.0 & 0.0 & .58326 & .49991 & -7.41638 & 0.0 & 0.0 & 1.24988 & -13.49996 & 24.06442 & 0.0 \\
1.24988 & -.66662 & .08335 & 0.0 & 0.0 & -13.49996 & 6.74998 & -.66662 & 0.0 & 0.0 & 25.47978 \\
.66662 & 1.16674 & -.66662 & .08335 & 0.0 & 0.0 & -13.49996 & 6.74998 & -.66662 & 0.0 & -13.49996 \\
.08335 & .66662 & 1.16674 & -.66662 & .08335 & .66662 & 0.0 & -13.49996 & 6.74998 & -.66662 & 1.16674 \\
0.0 & .08335 & .66662 & 1.16674 & -.66662 & 0.0 & .66662 & 0.0 & -13.49996 & 6.74998 & 0.0 \\
0.0 & 0.0 & .08335 & .66662 & 1.24988 & 0.0 & 0.0 & .66662 & 0.0 & -13.49996 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.24988 & -.66662 & .08335 & 0.0 & 0.0 & -13.49996 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & .66662 & 1.16674 & -.66662 & .08335 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & .08335 & .66662 & 1.16674 & -.66662 & .08335 & .66662 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & .08335 & .66662 & 1.16674 & -.66662 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & .08335 & .66662 & 1.24988 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.24988 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & .66662 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & .08335 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0
\end{array}\right.
$$

Equation (6.1) (Continued)

.66662	.08335	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.16674	.66662	.08335	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-.66662	1.16674	.66662	.08335	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.08335	-.66662	1.16674	.66662	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	.08335	-.66662	1.24988	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	.66662	0.0	0.0	1.24988	.66662	.08335	0.0	0.0	0.0	0.0
-13.49996	0.0	.66662	0.0	-.66662	1.16674	.66662	.08335	0.0	0.0	0.0
6.74998	-13.49996	0.0	.66662	.08335	-.66662	1.16674	.66662	.08335	0.0	0.0
-.66662	6.74998	-13.49996	0.0	0.0	.08335	-.66662	1.16674	.66662	0.0	0.0
0.0	-.66662	6.74998	-13.49996	0.0	0.0	.08335	-.66662	1.24988	0.0	0.0
-13.49996	1.16674	0.0	0.0	-13.49996	0.0	.66662	0.0	0.0	1.24988	.66662
26.99993	-13.49996	1.16674	0.0	6.74998	-13.49996	0.0	.66662	0.0	-.66662	1.16674
-13.49996	26.99993	-13.49996	1.16674	-.66662	6.74998	-13.49996	0.0	.66662	.08335	-.66662
1.16674	-13.49996	26.99993	-13.49996	0.0	-.66662	6.74998	-13.49996	0.0	0.0	.08335
0.0	1.16674	-13.49996	25.47978	0.0	0.0	-.66662	6.74998	-13.49996	0.0	0.0
6.74998	-.66662	0.0	0.0	25.47978	-13.49996	1.16674	0.0	0.0	-13.49996	0.0
-13.49996	6.74998	-.66662	0.0	-13.49996	26.99993	-13.49996	1.16674	0.0	6.74998	-13.49996
0.0	-13.49996	6.74998	-.66662	1.16674	-13.49996	26.99993	-13.49996	1.16674	-.66662	6.74998
.66662	0.0	-13.49996	6.74998	0.0	1.16674	-13.49996	26.99993	-13.49996	0.0	-.66662
0.0	.66662	0.0	-13.49996	0.0	0.0	1.16674	-13.49996	25.47978	0.0	0.0
-.66662	.08335	0.0	0.0	-13.49996	6.74998	.66662	0.0	0.0	24.06442	-13.49996
1.16674	-.66662	.08335	0.0	0.0	-13.49996	6.74998	-.66662	0.0	-13.49996	25.47978
.66662	1.16674	-.66662	.08335	.66662	0.0	-13.49996	6.74998	-.66662	1.24988	-13.49996
.08335	.66662	1.16674	-.66662	0.0	.66662	0.0	-13.49996	6.74998	0.0	1.24988
0.0	.08335	.66662	1.24988	0.0	0.0	.66662	0.0	-13.49996	0.0	0.0
0.0	0.0	0.0	0.0	1.24988	-.66662	.08335	0.0	0.0	-7.41638	4.16624
0.0	0.0	0.0	0.0	.66662	1.16674	-.66662	.08335	0.0	.49991	-7.33302
0.0	0.0	0.0	0.0	.08335	.66662	1.16674	-.66662	.08335	.58326	.49991
0.0	0.0	0.0	0.0	0.0	.08335	.66662	1.16674	-.66662	0.0	.58326
0.0	0.0	0.0	0.0	0.0	0.0	.08335	.66662	1.24988	0.0	0.0

Equation (6.1) (Continued)

$$
\begin{bmatrix}
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
.08335 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
.66662 & .08335 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
1.16674 & .66662 & .08335 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
-.66662 & 1.16674 & .66662 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
.08335 & -.66662 & 1.24988 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
.66662 & 0.0 & 0.0 & 1.24988 & .66662 & .08335 & 0.0 & 0.0 \\
0.0 & .66662 & 0.0 & -.66662 & 1.16674 & .66662 & .08335 & 0.0 \\
-13.49996 & 0.0 & .66662 & .08335 & -.66662 & 1.16674 & .66662 & .08335 \\
6.74998 & -13.49996 & 0.0 & 0.0 & .08335 & -.66662 & 1.16674 & .66662 \\
-.66662 & 6.74998 & -13.49996 & 0.0 & 0.0 & .08335 & -.66662 & 1.24988 \\
1.24988 & 0.0 & 0.0 & -7.41638 & .49991 & .58326 & 0.0 & 0.0 \\
-13.49996 & 1.24988 & 0.0 & 4.16624 & -7.33302 & .49991 & .58326 & 0.0 \\
25.47978 & -13.49996 & 1.24988 & -.58326 & 4.16624 & -7.33302 & .49991 & .58326 \\
-13.49996 & 25.47978 & -13.49996 & 0.0 & -.58326 & 4.16624 & -7.33302 & .49991 \\
1.24988 & -13.49996 & 24.06442 & 0.0 & 0.0 & -.58326 & 4.16624 & -7.24966 \\
-.58326 & 0.0 & 0.0 & 9.41664 & -5.35850 & .64562 & 0.0 & 0.0 \\
4.16624 & -.58326 & 0.0 & -5.35850 & 10.83330 & -5.35850 & .64562 & 0.0 \\
-7.33302 & 4.16624 & -.58326 & .64562 & -5.35850 & 10.83330 & -5.35850 & .64562 \\
.49991 & -7.33302 & 4.16624 & 0.0 & .64562 & -5.35850 & 10.83330 & -5.35850 \\
.58326 & .49991 & -7.24966 & 0.0 & 0.0 & .64562 & -5.35850 & 10.74995
\end{bmatrix}
\begin{bmatrix} W_1 \\ W_2 \\ W_3 \\ W_4 \\ W_5 \\ W_6 \\ W_7 \\ W_8 \\ W_9 \\ W_{10} \\ W_{11} \\ W_{12} \\ W_{13} \\ W_{14} \\ W_{15} \\ W_{16} \\ W_{17} \\ W_{18} \\ W_{19} \\ W_{20} \\ W_{21} \\ W_{22} \\ W_{23} \\ W_{24} \\ W_{25} \\ W_{26} \\ W_{27} \\ W_{28} \\ W_{29} \\ W_{30} \end{bmatrix}
=
\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \\ P_9 \\ P_{10} \\ P_{11} \\ P_{12} \\ P_{13} \\ P_{14} \\ P_{15} \\ P_{16} \\ P_{17} \\ P_{18} \\ P_{19} \\ P_{20} \\ P_{21} \\ P_{22} \\ P_{23} \\ P_{24} \\ P_{25} \\ P_{26} \\ P_{27} \\ P_{28} \\ P_{29} \\ P_{30} \end{bmatrix}
\frac{\Delta A}{D}
\qquad (6.1)
$$

3. <u>Deflection Influence Coefficients For Basic Panel</u>. The inverse of the coefficients matrix on the left side gives the deflection influence coefficients for the plate, which are presented in Table 6.1.

TABLE 6.1

η_{ij} Deflection Influence Coefficients for Plate

i \ j	1	2	3	4	5	6	7	8	9	10
1	.20033	.18446	.14633	.10477	.02937	.13342	.14894	.12578	.09347	.02472
2	.18491	.21669	.18920	.14482	.08571	.15375	.19063	.18045	.12534	.06148
3	.14730	.18676	.23607	.17814	.14007	.09707	.16178	.21263	.16779	.10683
4	.10631	.14649	.17951	.20495	.18129	.05803	.12121	.17398	.18582	.13617
5	.02889	.08674	.14175	.18024	.19005	.02302	.07479	.10816	.14072	.14389
6	.13861	.15515	.09758	.05778	.02311	.26963	.25885	.18473	.14006	.06055
7	.14997	.19143	.16435	.12018	.07544	.26116	.29583	.28247	.17502	.14474
8	.12634	.18162	.21411	.17456	.10842	.18767	.27743	.36516	.26808	.19829
9	.09105	.12475	.16999	.18715	.14175	.14190	.17405	.26893	.30969	.26809
10	.02414	.06179	.10734	.13711	.14483	.06111	.14282	.19466	.26481	.27734
11	.11648	.14173	.07437	.04453	.02342	.23418	.22572	.14474	.09338	.04108
12	.12069	.15436	.13333	.09199	.06625	.19516	.22141	.18443	.10272	.09804
13	.10308	.12295	.16943	.11499	.10117	.14161	.16492	.23158	.17462	.14941
14	.07236	.08963	.11553	.13821	.12172	.08931	.09872	.18209	.22504	.21477
15	.01900	.04012	.06677	.14482	.12578	.03846	.04669	.15306	.22178	.24106
16	.08942	.08597	.06061	.03066	.01617	.18541	.17514	.14078	.07704	.02929
17	.09239	.08875	.06480	.04777	.05131	.15265	.18432	.16248	.09187	.06630
18	.07564	.07627	.08551	.07332	.08782	.11309	.12383	.20030	.12171	.10683
19	.04724	.05238	.06986	.08729	.09845	.06929	.09059	.15873	.17616	.14909
20	.01485	.02455	.06564	.08779	.09455	.02767	.07501	.13562	.16855	.19260
21	.05706	.05162	.04298	.01808	.00619	.13355	.10329	.09130	.05679	.01954
22	.05803	.05546	.04761	.02928	.02572	.10272	.12391	.11400	.06061	.04206
23	.04576	.04622	.05837	.04931	.04775	.07647	.08927	.13587	.08777	.07194
24	.02415	.02820	.04832	.05540	.05670	.08674	.06153	.08838	.12298	.10221
25	.00462	.01643	.04880	.05054	.05976	.01746	.04131	.07709	.10334	.13296
26	.01708	.01817	.01279	.00618	.00123	.06023	.05743	.04853	.00359	.00606
27	.02737	.02805	.01849	.01694	.00628	.05131	.05598	.04826	.03004	.01910
28	.01946	.02032	.01951	.01835	.01232	.04775	.04738	.05731	.04622	.04160
29	.00523	.00868	.02034	.02690	.01828	.01620	.02769	.04679	.05556	.05238
30	.00176	.00608	.01398	.01747	.01306	.00479	.01467	.02622	.04879	.05602

TABLE 6.1. (Continued)

η_{ij} Deflection Influence Coefficients for Plate										
i \ j	11	12	13	14	15	16	17	18	19	20
1	.11742	.12126	.10225	.07257	.01952	.08839	.09300	.07542	.04786	.01477
2	.14020	.15315	.12172	.08834	.03901	.08579	.08835	.07706	.05276	.07107
3	.07497	.13173	.16845	.11453	.06777	.06166	.06563	.08679	.07086	.06486
4	.04546	.09106	.11452	.13728	.14381	.03052	.04950	.07165	.08786	.08837
5	.02311	.06517	.10014	.12275	.09586	.01541	.05077	.08647	.09755	.09549
6	.23250	.19413	.14007	.08837	.03806	.19000	.15407	.11240	.06827	.02772
7	.22495	.22099	.16584	.09967	.04765	.17373	.18489	.12323	.07807	.03659
8	.14343	.18396	.23252	.18302	.15397	.13868	.16178	.19410	.15713	.13409
9	.09655	.10365	.17359	.22598	.22094	.07536	.09012	.12013	.17702	.16643
10	.04160	.09898	.14875	.21337	.23998	.03059	.06749	.10817	.14789	.19046
11	.36666	.37606	.31683	.17667	.08258	.26812	.24314	.18029	.07623	.04395
12	.32356	.46850	.41601	.28565	.16139	.24496	.36055	.32175	.18509	.11557
13	.26959	.37235	.53927	.36978	.27272	.18955	.19785	.41603	.28333	.18491
14	.15921	.28361	.40624	.46931	.33280	.12015	.18254	.31889	.36976	.24033
15	.08964	.17970	.30655	.37352	.37720	.04577	.07810	.17564	.24653	.27736
16	.27734	.24652	.17564	.07812	.04576	.37722	.37349	.30655	.17973	.08966
17	.24036	.36978	.31889	.18258	.12018	.33281	.46933	.40624	.28363	.15922
18	.18489	.28335	.41601	.19783	.18951	.27270	.36979	.53926	.37238	.26957
19	.11555	.18507	.32171	.36054	.24498	.16138	.28567	.41603	.46847	.32357
20	.04391	.07626	.18027	.24313	.26809	.08259	.17669	.31682	.37609	.36660
21	.19044	.14791	.10816	.06748	.03051	.23996	.21334	.14876	.09896	.04162
22	.16640	.17703	.12018	.09013	.07534	.22094	.22596	.17359	.10367	.09657
23	.13404	.15716	.19413	.16178	.13867	.15396	.18298	.23535	.18399	.14341
24	.03655	.07805	.12327	.18489	.17375	.04764	.09963	.16588	.22095	.22493
25	.02773	.06825	.11242	.15407	.19003	.03809	.08836	.14010	.19410	.23452
26	.09549	.09757	.08643	.05079	.01539	.09590	.12277	.10017	.06519	.02313
27	.08833	.08786	.07164	.04946	.03050	.14380	.13729	.11453	.09102	.04545
28	.06489	.07086	.08676	.06561	.06167	.06775	.11455	.16846	.13176	.07497
29	.07100	.05274	.07704	.08837	.08578	.03903	.08836	.12174	.15319	.14023
30	.01479	.04782	.07544	.09301	.08839	.01950	.07259	.10226	.12121	.11740

TABLE 6.1. (Continued)

η_{ij} Deflection Influence Coefficients for Plate											
i \ j	21	22	23	24	25	26	27	28	29	30	$\sum$
1	.05604	.04877	.02623	.01466	.00477	.01309	.01747	.01399	.00609	.00177	2.14722
2	.05239	.05555	.04677	.02770	.01622	.01829	.02692	.02033	.00869	.00524	2.72838
3	.04162	.04626	.05733	.04740	.04777	.01230	.01837	.01954	.02034	.01949	2.87190
4	.01910	.03003	.04829	.05596	.05133	.00626	.01699	.01852	.02809	.02735	2.65550
5	.00607	.00358	.04855	.05740	.06022	.00122	.00617	.01281	.01819	.01707	2.10234
6	.13296	.10331	.07713	.04129	.01748	.05977	.05055	.04882	.01645	.00460	3.04393
7	.10223	.12296	.08839	.06151	.08676	.05673	.05542	.04829	.02817	.02411	3.72450
8	.07196	.08775	.13586	.08926	.07649	.04773	.04933	.05833	.04624	.04579	4.26970
9	.04207	.06066	.11396	.12393	.10276	.02574	.02929	.04766	.05545	.05800	3.98654
10	.01956	.05677	.09132	.10326	.13359	.00621	.01806	.04299	.05166	.05709	3.28353
11	.19263	.16853	.13563	.07506	.02769	.09453	.08781	.06563	.02457	.01482	4.15692
12	.14913	.17619	.15877	.09058	.06927	.09844	.08732	.06987	.05241	.04726	5.25087
13	.10686	.12173	.20033	.12386	.11312	.08780	.07335	.08552	.07629	.07566	5.63338
14	.06634	.09189	.16249	.18433	.15267	.05133	.04779	.06482	.08877	.09242	5.23298
15	.02925	.07706	.14077	.17516	.18543	.01616	.03063	.06063	.08595	.08943	4.13802
16	.24110	.22174	.15309	.04669	.03847	.12578	.14484	.06673	.04014	.01906	4.13801
17	.21474	.22509	.18213	.09873	.08934	.12174	.13823	.11555	.08966	.07238	5.23293
18	.14943	.17466	.23162	.16493	.14163	.10119	.11502	.16946	.12297	.10309	5.63326
19	.09807	.10276	.18440	.22144	.19513	.06623	.09196	.13330	.15438	.12072	5.25096
20	.04109	.09343	.14471	.22573	.23420	.02345	.04457	.07439	.14170	.11649	4.15681
21	.27730	.26486	.19460	.14284	.06116	.14486	.13710	.10736	.06177	.02416	3.28370
22	.26810	.30963	.26896	.17406	.14192	.14179	.18712	.17004	.12473	.09106	3.98650
23	.19826	.26811	.36519	.27744	.18769	.10844	.17459	.21414	.18160	.12631	4.26983
24	.14477	.17504	.28241	.29588	.26118	.07542	.12019	.16432	.19141	.14999	3.72464
25	.06051	.14009	.18476	.25881	.26966	.02316	.05775	.09762	.15515	.13863	3.04399
26	.14387	.14075	.10811	.07474	.02304	.19003	.18025	.14177	.08673	.02892	2.10243
27	.13616	.18588	.17395	.12121	.05806	.18132	.20495	.17953	.14649	.10634	2.65555
28	.10684	.16776	.21266	.16178	.09709	.14009	.17815	.23606	.18676	.14732	2.87199
29	.06149	.12535	.18047	.19062	.15377	.08573	.14480	.18920	.21670	.18493	2.72863
30	.02474	.09348	.12580	.14893	.13346	.02939	.10479	.14630	.18449	.20031	2.14746

TABLE 6.2

η_{ij} Deflection Influence Coefficients for Beam					
i \ j	1	2	3	4	5
1	1.388888	2.11111	2.16667	1.72222	.94444
2	2.11111	3.55555	3.83333	3.11111	1.72222
3	2.16667	3.83333	4.50000	3.83333	2.16667
4	1.72222	3.1111	3.83333	3.55555	2.11111
5	.94444	1.72222	2.16667	2.11111	1.38888

4. Deflection Influence Coefficients For Beam. Table 6.2 represents deflection influence coefficients for a basic beam calculated by statics.

6.3. Numerical Example. A two-span continuous skew plate-beam structure is analyzed for uniformly distributed load (Figure 6.2) making use of the deflection influence coefficients for basic plate and basic beam obtained in article 6.2. Each span is covered by a skew network. The relative stiffness of edge beam to the plate $\frac{EI}{D}$ is assumed to be equal to two. In the solution of the problem all values unless otherwise stated, are in kips, feet and kip-feet. References to the equations and tables used are given in the example.

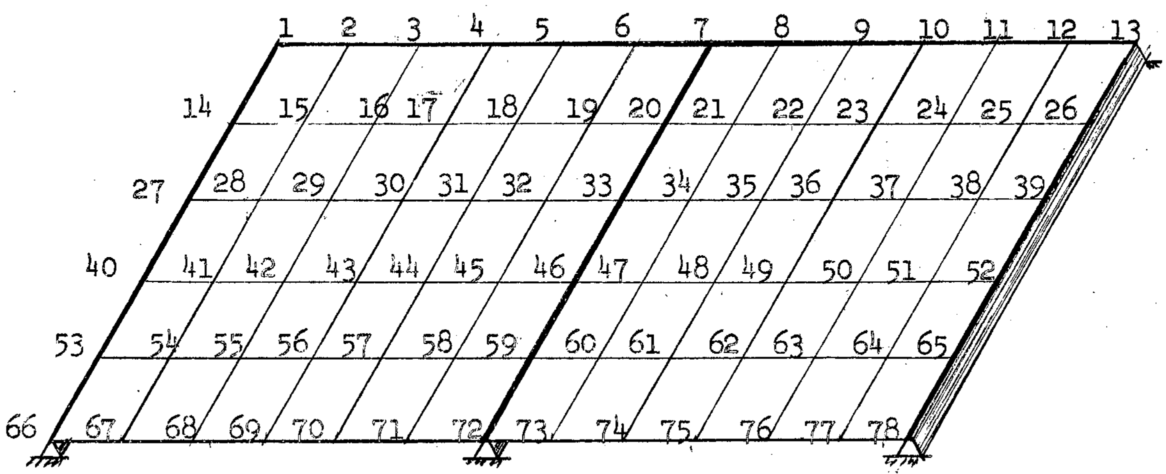

Figure 6.2. Two-Span Continuous Skew Plates

1. Angular and Displacement Functions. Angular flexibilities and angular carry-over values (F,G) are obtained by susbtituting the values of η_{ij} from Table 6.1. in the expressions (2.5), (3.6) and (3.9).

$$\sum F_{7,7} = \sum F_{72,72} = .33808\,\frac{1}{D}$$
$$\sum F_{20,20} = \sum F_{59,59} = .47369\,\frac{1}{D}$$
$$\sum F_{33,33} = \sum F_{46,46} = .64420\,\frac{1}{D}$$

TABLE 6.3

G_{ij} - Angular Carry-Over

i \ j	7	20	33	46	59	72
7	—	.24016	.18470	.15924	.10068	.02612
20	.24019	—	.40918	.32949	.23084	.10068
33	.18470	.41160	—	.41163	.32946	.15926
46	.15920	.32948	.41166	—	.41157	.18473
59	.10067	.23085	.32950	.40921	—	.24020
72	.02612	.10069	.15922	.18470	.24018	—

Angular load functions (τ) are calculated using expressions (2.5) and (3.3) and Table 6.1.

$$\sum\tau_7 = \sum\tau_{72} = 99.66754 \ \frac{1}{D}$$

$$\sum\tau_{20} = \sum\tau_{59} = 148.63634 \ \frac{1}{D}$$

$$\sum\tau_{33} = \sum\tau_{46} = 194.54632 \ \frac{1}{D}$$

Displacement-angular carry-overs (Q) are obtained by using expressions (2.5) and (3.13). They are presented in Table 6.4.

TABLE 6.4

Q_{ji} - Displacement-Angular Carry-Over

i \ j	7	20	33	
2	.12510	.10453	.08227	77
3	.37559	.26756	.17372	76
4	.61379	.46479	.28912	75
5	.78046	.59370	.62709	74
6	.82294	.62717	.54464	73
8	.86745	.60020	.50437	71
9	.79873	.67182	.61371	70
10	.63362	.42253	.32203	69
11	.45366	.25019	.10283	68
12	.12717	.10007	.10141	67
67	.00528	.02689	.06997	12
68	.02672	.07820	.13263	11
69	.05547	.18615	.26253	10
70	.07876	.22369	.37217	9
71	.07399	.24720	.38724	8
73	.05668	.25881	.40932	6
74	.07565	.21889	.38023	5
75	.06056	.21139	.28418	4
76	.02637	.07123	.10639	3
77	.00766	.01992	.06417	2
	72	59	46	j \ i

Displacement load functicns (δ) are obtained from expression (3.10)

$$\sum\delta_2 = \sum\delta_8 = 251.68279 \ \frac{1}{D}$$

$$\sum\delta_3 = \sum\delta_9 = 319.72804 \ \frac{1}{D}$$

$$\sum\delta_4 = \sum\delta_{10} = 336.54658 \ \frac{1}{D}$$

$$\sum\delta_5 = \sum\delta_{11} = 311.18752 \ \frac{1}{D}$$

$$\sum\delta_6 = \sum\delta_{12} = 246.36489 \ \frac{1}{D}$$

Displacement flexibilities (D) are calculated using expressions (2.9), (3.11) and (4.8).

$$D_{2,2} = D_{8,8} = 7.23076 \ \frac{1}{D}$$

$$D_{3,3} = D_{9,9} = 12.09886 \ \frac{1}{D}$$

$$D_{4,4} = D_{10,10} = 14.48603 \ \frac{1}{D}$$

$$D_{5,5} = D_{11,11} = 11.84468 \ \frac{1}{D}$$

$$D_{6,6} = D_{12,12} = 7.00820 \ \frac{1}{D}$$

Displacement carry-overs(H) are obtained using expressions (2.9), (3.12) and (4.9). They are presented in Table 6.5.

The general matrix is formed using equation (2.10) in terms of angular, displacement and load functions. This is presented on Page 52.

TABLE 6.5

H_{ij} Displacement Carry-Overs										
i \ j	2	3	3	5	6	67	68	69	70	71
2	—	8.39180	7.68201	5.85628	2.60346	.28340	.37823	.30289	.13185	.03832
3	8.40154	—	12.08238	9.61690	5.44362	.39599	.58283	.44015	.18814	.11345
4	7.70301	12.07156	—	11.82493	7.54647	.26630	.39772	.42305	.44037	.42197
5	5.88962	9.62306	11.84259	—	8.11867	.13553	.36784	.40097	.60816	.59217
6	2.59307	5.46592	7.56285	8.20044	—	.02641	.13358	.27734	.39382	.36957

$$
\begin{bmatrix}
.36420 & .68168 & .68784 & .26532 & .80392 & 1.34870 & 1.71222 & 1.75942 & 1.92288 & 1.75498 & 1.37818 & .96076 & .26490 \\
.34087 & 1.34906 & 1.47734 & .24890 & .67758 & 1.35236 & 1.62518 & 1.77196 & 1.69480 & 1.79102 & 1.21736 & .65678 & .25392 \\
.34396 & 1.48212 & 2.11166 & .29288 & .56022 & 1.14660 & 2.01464 & 1.90792 & 1.78322 & 1.97168 & 1.16912 & .65090 & .34276 \\
.13276 & .24890 & .29288 & 14.46152 & 16.80388 & 15.40602 & 11.77924 & 5.18614 & .07664 & .22690 & .84394 & 1.18254 & .73914 \\
.40196 & .67758 & .56022 & 16.80388 & 24.19772 & 24.14312 & 19.24612 & 10.93184 & .26370 & .37628 & .88074 & 1.21632 & .78764 \\
.67435 & 1.35236 & 1.14660 & 15.40602 & 24.14312 & 28.97206 & 23.68518 & 15.12570 & .60578 & .88030 & .84610 & .80194 & .55468 \\
.85611 & 1.62518 & 2.01464 & 11.77924 & 19.24612 & 23.68518 & 23.68930 & 16.40088 & .75646 & 1.16566 & .79554 & .73568 & .26716 \\
.87992 & 1.77196 & 1.90792 & 5.18614 & 10.93184 & 15.12570 & 16.40088 & 14.01640 & .56680 & .79198 & .53260 & .27106 & .05282 \\
.96144 & 1.69480 & 1.78322 & .07664 & .26370 & .60578 & .75646 & .56680 & 14.46152 & 16.80388 & 15.40602 & 11.77924 & 5.18614 \\
.87744 & 1.79102 & 1.97168 & .22690 & .37628 & .88030 & 1.16566 & .79198 & 16.80388 & 24.14772 & 24.14312 & 19.24612 & 10.93184 \\
.86909 & 1.21736 & 1.16912 & .84394 & .88074 & .84610 & .79554 & .53260 & 15.40602 & 24.14312 & 28.97206 & 23.68518 & 15.12570 \\
.48038 & .65678 & .65090 & 1.18254 & 1.21632 & .80194 & .73568 & .27106 & 11.77924 & 19.24612 & 23.68518 & 23.68930 & 16.40088 \\
.13245 & .25392 & .34276 & .73914 & .78764 & .55468 & .26712 & .05282 & 5.18614 & 10.93184 & 15.12570 & 16.40088 & 14.01640
\end{bmatrix}
\begin{bmatrix}
M_7 \\ M_{20} \\ M_{33} \\ -R_2 \\ -R_3 \\ -R_4 \\ -R_5 \\ -R_6 \\ -R_8 \\ -R_9 \\ -R_{10} \\ -R_{11} \\ -R_{12}
\end{bmatrix}
+
\begin{bmatrix}
39.86702 \\ 59.45454 \\ 77.81853 \\ 100.67312 \\ 127.89122 \\ 134.61863 \\ 124.47310 \\ 98.54596 \\ 100.67312 \\ 127.89122 \\ 134.61863 \\ 124.47310 \\ 98.54596
\end{bmatrix}
= 0
$$

The solution of this matrix yields to the final redundants, which are,

$$
\begin{aligned}
M_7 &= -21.09870 \\
M_{20} &= -18.21344 \\
M_{33} &= -15.16309 \\
R_2 &= 0.75500 \\
R_3 &= 1.02690 \\
R_4 &= 1.30069 \\
R_5 &= 0.96060 \\
R_6 &= 0.72632 \\
R_8 &= 0.74075 \\
R_9 &= 0.98600 \\
R_{10} &= 1.25333 \\
R_{11} &= 0.93005 \\
R_{12} &= 0.70638
\end{aligned}
$$

CHAPTER VII

SUMMARY AND CONCLUSIONS

7.1. Summary. The application of the flexibility methods to the analysis of continuous skew plate-beam systems is presented. The continuous structure is isolated into appropriate basic structures, and the support moments and edge shears are selected as the redundants. Angular, displacement and load functions of the basic structure are introduced and expressed in terms of the deflection influence coefficients for a basic skew panel and basic beam. Deformation equations in terms of the redundants and these functions are obtained utilizing the conditions of compatibility of deformations over a continuous support and between the plate and the beam. The procedure for the analysis of the problem is outlined and a numerical example is included.

7.2. Conclusions. The use of flexibility methods to formulate the problems of one-way continuous skew plate beam systems permits the selection of moments over supports and edge shears as redundants and affords a significant reduction in the number of unknowns in a given problem. The general technique requires the availibility of tables of influence coefficients for basic skew panels. One such table, based on a thirty unit finite difference network, is applied in this thesis to a numerical example. Similar tables can be readily evaluated by matrix inversion for other length-width ratios and angles of skew. If higher accuracy is required, a finer difference network can be used.

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